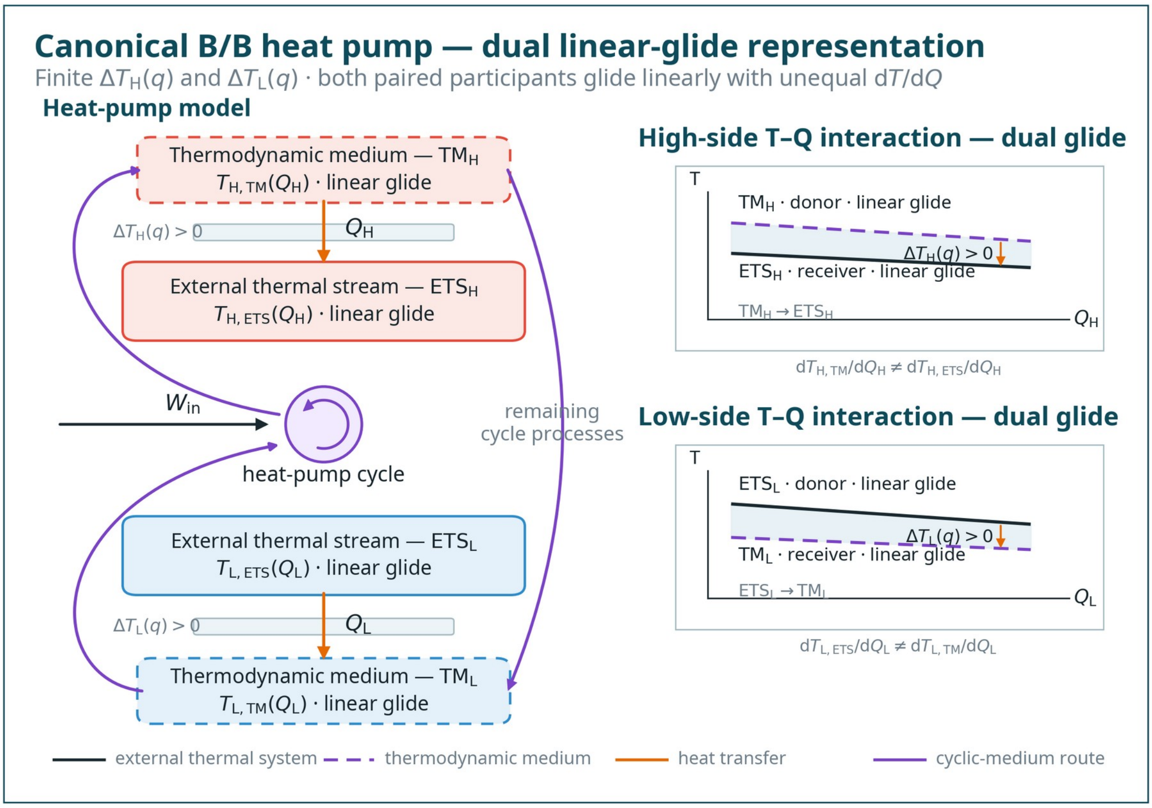
APPLICATIONS & RESEARCH OPPORTUNITIES · TECHNICAL COMPANION

Thermal Matching in Non-Isothermal Thermodynamic Cycles

Paired-Interface Modelling Demonstrated with Heat Pumps



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Contents

A controlled reader-facing route through the paired-interface modelling method.

NAVIGATION NOTE
Chapters 6–8 use the same page architecture: model definition, governing temperature trajectories, paired-interface figure and qualification note. This controlled repetition makes the three heat-pump cases directly comparable.

Report at a Glance

Many familiar heat-engine, heat-pump and refrigeration schematics show a high-temperature external reservoir, a cycle symbol and a low-temperature external reservoir. That compact representation is highly effective for energy-flow logic and for the classical two-isothermal idealisation, but it commonly leaves the thermodynamic-medium heat-transfer branches implicit. The two thermal interfaces therefore appear as two reservoir temperatures rather than as four explicitly owned boundary-temperature trajectories: the external thermal system and the thermodynamic medium on the high side, and the external thermal system and the thermodynamic medium on the low side.

That hidden structure becomes especially important as soon as at least one of those four trajectories is non-isothermal. A finite-capacity source or sink may heat or cool through the exchanger; a thermodynamic medium may glide during phase change or follow another non-isothermal heat-addition or heat-rejection path. The relevant design question is then not only how large the local temperature difference is, but how the two paired T-Q trajectories evolve relative to one another over the complete transferred-heat coordinate. This report makes that pairing explicit and uses it to illustrate Thermal Matching, with non-isothermal trajectory matching as its principal worked focus.

Paired-interface modelling is the engineering focus of this companion. The broader CarnotX programme links such non-isothermal interactions to the operationalized Clausius reversible-cycle equality. The classical Carnot framework exactly defines the corresponding reversible cycle within the two-isothermal case and therefore supports direct comparison of an irreversible isothermal cycle with its corresponding reversible isothermal cycle. For non-isothermal heat interactions, the standard two-isothermal Carnot formulation does not itself define the corresponding process-specific reversible non-isothermal cycle from the actual heat–temperature interactions; the proposed operationalization of the Clausius equality developed within the CarnotX programme provides the broader cycle-to-cycle representation used here. Formal benchmark-relative lost-work and excess-work quantification remains in the Formal Core.

CENTRAL DISTINCTION
Thermal Matching is not restricted to non-isothermal cycles. In a fully isothermal heat-transfer interaction, the two paired trajectories may be horizontal yet remain separated by a finite constant temperature difference. Deliberately changing one or both temperature levels can constitute Thermal Matching when it improves their local alignment without relying solely on increased heat-transfer conductance; if the reduction results solely from increasing U or UA, it is conductance enhancement. Non-isothermal interactions add a further design degree of freedom because the trajectory shapes themselves may also be mismatched and adapted. The title of this companion reflects its principal non-isothermal focus, not an exclusivity condition for the method.

Three canonical cases

Case	Low-side external system	Low-side thermodynamic medium	Visible result
Fully isothermal	Constant at 10 °C	Constant at 5 °C	Constant 5 K approach; matched profile shapes but finite separation
External-source glide	20 → 10 °C	Constant at 5 °C	Linear-versus-horizontal trajectory mismatch
TM glide	Constant at 10 °C	-5 → 5 °C	Horizontal-versus-linear trajectory mismatch

The stated temperatures are illustrative and were selected to preserve the 10 K high-side approach and 5 K minimum low-side approach used in the working examples. They do not constitute a calculated refrigerant design or validated equipment model.

What This Report Does - and Does Not Do

PURPOSE AND DISCLOSURE BOUNDARY
This public technical companion develops one focused engineering application: making Thermal Matching visible and operational through paired heat-transfer-interface representations. It sits within the same publication programme as the broader Impact, Applications & Research Opportunities Report, but it addresses a more local engineering question and can be read without the formal reversible-cycle construction. Its starting point is a representation issue: conventional reservoir-cycle-reservoir diagrams usually show the external hot and cold thermal systems while leaving the corresponding thermodynamic-medium heat-transfer trajectories inside the cycle symbol. When one or more of the four interface trajectories is non-isothermal, that omission can hide a temperature-profile mismatch that matters for heat-transfer design and cycle performance. This report exposes those paired trajectories, distinguishes their shapes from conductance effects, and illustrates how they may be adapted. It is not a complete heat-pump design method, a replacement for the Formal Core, or a claim that temperature-profile matching itself is historically new.

What it does

- Defines the thermodynamic medium and external thermal system as separate physical participants at each heat-transfer interface.
- Standardises energy-flow direction, boundary-temperature labels, T-Q orientation, colours, line styles and minimum model annotations.
- Documents three canonical illustrative heat-pump cases and the assumptions needed to compare them.
- Adds a dual-glide engineering synthesis in which Thermal Matching is improved simultaneously at the high- and low-temperature interfaces, without identifying the improved configuration as the thermodynamic optimum or reversible limit.
- Uses the heat-engine directional counterpart as the Chapter 3 conceptual bridge, then applies the same visual grammar to heat-pump architectures in Chapter 4 without changing the report’s three canonical heat-pump cases.
- Separates conductance enhancement from Thermal Matching, and distinguishes isothermal temperature-level matching from non-isothermal trajectory-shape matching, while keeping both separate from other internal sources of cycle irreversibility.
- Separates interface-level quantification of Thermal Matching and Thermal Mismatching from cycle-level quantification of the associated thermodynamic consequence; the latter remains part of the Formal Core method.
- Provides a reusable paired-interface protocol for isothermal, piecewise-isothermal, continuously non-isothermal and mixed cases, with Thermal Matching assessed under a declared comparison basis.

What it does not do

- It does not prove that the compressor, expansion device, flow arrangement or refrigerant state path is reversible or technically realizable.
- It does not infer complete-cycle reversibility from zero temperature difference at the two external heat-transfer interfaces.
- It does not treat one zero terminal pinch as proof of process-wide reversible heat transfer.
- It does not assume that increasing U or the heat-transfer conductance UA changes temperature profiles unless the comparison class permits the operating state, flow, area, duty or terminal temperatures to change.
- It does not use the local finite temperature difference of one heat-transfer process as a standalone scalar measure of the thermodynamic magnitude of irreversibility. Interface-level assessment characterises where and how thermal mismatch occurs; cycle-level work consequence is quantified separately through the complete-cycle actual-to-reversible comparison.

01 | Purpose, Scope and Scientific Status

A technical companion for modelling and visual communication, not a replacement for the Formal Core.

1.1 Primary purpose

The primary purpose is to make the physical heat-transfer pairing explicit where Thermal Matching can be assessed. At each interface, the reader must be able to identify the heat donor, the heat receiver, the temperature trajectory of the thermodynamic medium, the temperature trajectory of the external thermal system and the direction of the heat quantity. The paired-interface view is especially consequential when at least one participant changes temperature through the heat-transfer process, because trajectory-shape mismatch then becomes an additional design variable. The fully isothermal case is retained as a baseline and may still admit Thermal Matching through deliberate temperature-level alignment that is distinct from conductance enhancement.

1.2 Scope

The worked demonstration is a closed heat-pump cycle with one net low-temperature heat-receiving branch, one net high-temperature heat-supplying branch and net work input. Only linear T(Q) behaviour is considered for the non-isothermal illustrative cases. This corresponds to a constant effective dT/dQ over the stated interval, as can approximate a single-phase water stream with approximately constant heat-capacity rate under a bounded model. The fully isothermal case establishes the reference condition in which no profile-shape mismatch exists; Canonical Cases II and III then introduce one non-isothermal participant at the low-side interface. Chapter 3 uses an isothermal heat-engine counterpart to establish the paired-interface principle; Chapter 4 transfers the same visual grammar to the heat pump, which is the detailed demonstration used throughout the remainder of the report.

1.3 Epistemic status

Layer	Status in this report
Established science / engineering context	First Law; heat transfer through finite temperature difference is irreversible; local equality is required for reversible heat transfer; COP definitions. T-Q temperature-profile analysis and temperature-glide matching are established in specialist heat-exchanger and zeotropic heat-pump literature.
Formal Core interpretation	Formal Core interpretation: Carnot is treated as the exact two-isothermal specialization, while the operationalized Clausius reversible-cycle equality is proposed as the parent framework for constructing corresponding reversible cycles across admissible non-isothermal configurations. This companion uses that positioning but does not reproduce the formal construction or lost-work method.
Proposed operational method	Progressive reservoir-only, A/A and B/B visual grammar, together with a Thermal Matching method aligned with the Formal Core: improved local alignment may involve trajectory shape, temperature level, heat-duty distribution, or combinations of these, and is distinct from conductance enhancement. The A/A and B/B codes are report-specific standardised reporting conventions.
Illustrative calculation	The selected 60/50 °C high side and 20/10/5/-5 °C low-side temperatures.
Not established here	Technical realizability, equipment performance, optimized refrigerant composition and independently validated gains.

02 | System Boundary, Entities and Directions

One cycling medium; two external thermal interactions; two paired interfaces.

2.1 Investigated cycle

The investigated cyclic system is the thermodynamic medium. The labels TMH and TML identify its high- and low-temperature heat-transfer stages; they do not denote two separate media. The external high-temperature sink and low-temperature source remain outside that cyclic-system boundary.

2.2 External thermal system as umbrella term

External thermal system (ETS) is used as the umbrella term. An ideal isothermal thermal reservoir is one special case. A flowing water stream whose boundary temperature changes through the heat exchanger is more accurately described as a non-isothermal external thermal stream or finite-capacity thermal source. Canonical Case II is therefore referred to as External-Source Glide.

2.3 Energy directions

$$\mathrm{ETS}_L \xrightarrow{Q_L} \mathrm{TM}_L \rightarrow \mathrm{cycle} \rightarrow \mathrm{TM}_H \xrightarrow{Q_H} \mathrm{ETS}_H$$

At the low-temperature interface, the external thermal source is the donor and the thermodynamic medium is the receiver. At the high-temperature interface, the thermodynamic medium is the donor and the external thermal sink is the receiver. Net work enters the cycle.

$$Q_H = Q_L + W_{\mathrm{in}} \quad (1)$$

BOUNDARY RULE
Every temperature shown in a heat-transfer statement belongs to one declared physical side of the interface. A bulk stream temperature is used only when it adequately represents the boundary temperature for the stated model.

2.4 Local heat coordinate

Each interface uses its own cumulative positive heat magnitude Qj and normalized coordinate qj = Qj/Qj,tot. Both paired temperature trajectories on one interface are plotted against that same coordinate. High- and low-side plots may use the same normalized horizontal scale while representing different total heat quantities; QH and QL must not be visually or algebraically conflated.

$$q_j = \frac{Q_j}{Q_{j,\mathrm{tot}}}, 0 \le q_j \le 1 \quad (2)$$

2.5 Donor-receiver temperature difference

$$\Delta T_j(q) = T_{\mathrm{donor},j}(q) - T_{\mathrm{receiver},j}(q) \ge 0 \quad (3)$$

Equation (3) is used as a direct local diagnostic. The report does not collapse a varying gap into a universal scalar Thermal Matching index. Minimum, maximum and terminal approaches may be stated, but none alone represents the complete trajectory.

2.6 Terminology discipline

- Process statement: 'the heat-transfer process is thermodynamically irreversible because a finite local boundary-temperature difference is present.'
- Cycle statement: 'the complete cycle is irreversible' only after all relevant thermal and non-thermal mechanisms are included.
- Zero statement: identify the exact quantity, interface, interval and assumptions; do not infer absence of cycle irreversibility from cyclic state restoration.
- Energy-degradation language may be used as qualitative interpretation, but a quantitative value must name its defined method, boundary and comparison class.

03 | Heat-Engine Counterpart and Paired-Interface Principle

Chapter 3 uses the isothermal heat engine as the directional counterpart through which the paired-interface principle is introduced. Its heat quantities can be read in the natural top-to-bottom direction at both thermal interfaces: TRH transfers QH to TMH, and TML rejects QL to TRL, while Wout leaves the cycle. The purpose is conceptual: to expose temperature ownership, the thermodynamic-medium branches and the relationship between the A/A and B/B organisations before the same visual grammar is applied to the heat pump in Chapter 4.

3.1 Heat-engine counterpart progression

Representation	Visible information and diagnostic consequence
Heat engine - compact traditional bridge	Begins with the familiar reservoir-cycle-reservoir energy-flow sketch. The medium-side branches remain implicit on the left, while paired T-Q plots make their boundary temperatures visible.
Heat engine - explicit A/A, zero ΔT	Expands TMH and TML as separate stages of one cyclic medium. Coincident horizontal trajectories establish the reversible heat-transfer limit at both interfaces.
Heat engine - finite ΔT; A/A = B/B	Places each donor above its receiver at both interfaces: TRH → TMH and TML → TRL. Because both heat flows are downward, the energy-flow and paired-interface object orders coincide in this special isothermal case. The 10 K and 5 K gaps make both transfers irreversible.

3.2 Visual semantics and transferability

- Coral family: high-temperature interaction; blue family: low-temperature interaction; magenta: cycle device or cyclic-medium route.
- Heat-engine reading: TRH → TMH and TML → TRL; Wout leaves the cycle. In the finite-ΔT isothermal figure, donor-above-receiver ordering makes A/A and B/B coincide as a physical object arrangement. This is a special-case coincidence, not a general equivalence.
- Transfer to Chapter 4: for the heat pump, TRL → TML and TMH → TRH while Win enters the cycle. The reversed heat directions make the difference between A/A energy-flow organisation and B/B paired-interface organisation especially visible.
- Horizontal T-Q trajectories denote isothermal behaviour. Coincident trajectories represent the reversible heat-transfer limit; separated trajectories represent finite-temperature-difference heat transfer. Complete-cycle reversibility additionally requires every remaining cycle process to be reversible.

PROPOSED-METHOD STATUS The reservoir-only sketch is a conventional pedagogical abstraction. A/A and B/B are report-specific architecture codes: A/A denotes an explicit energy-flow organisation and B/B an explicit paired-interface organisation. The equality A/A = B/B stated for Figure 3 concerns only the object ordering of the displayed finite-ΔT isothermal heat-engine case; it is not a universal identity or established international nomenclature.

3.3 Traditional Compact Heat-Engine Representation

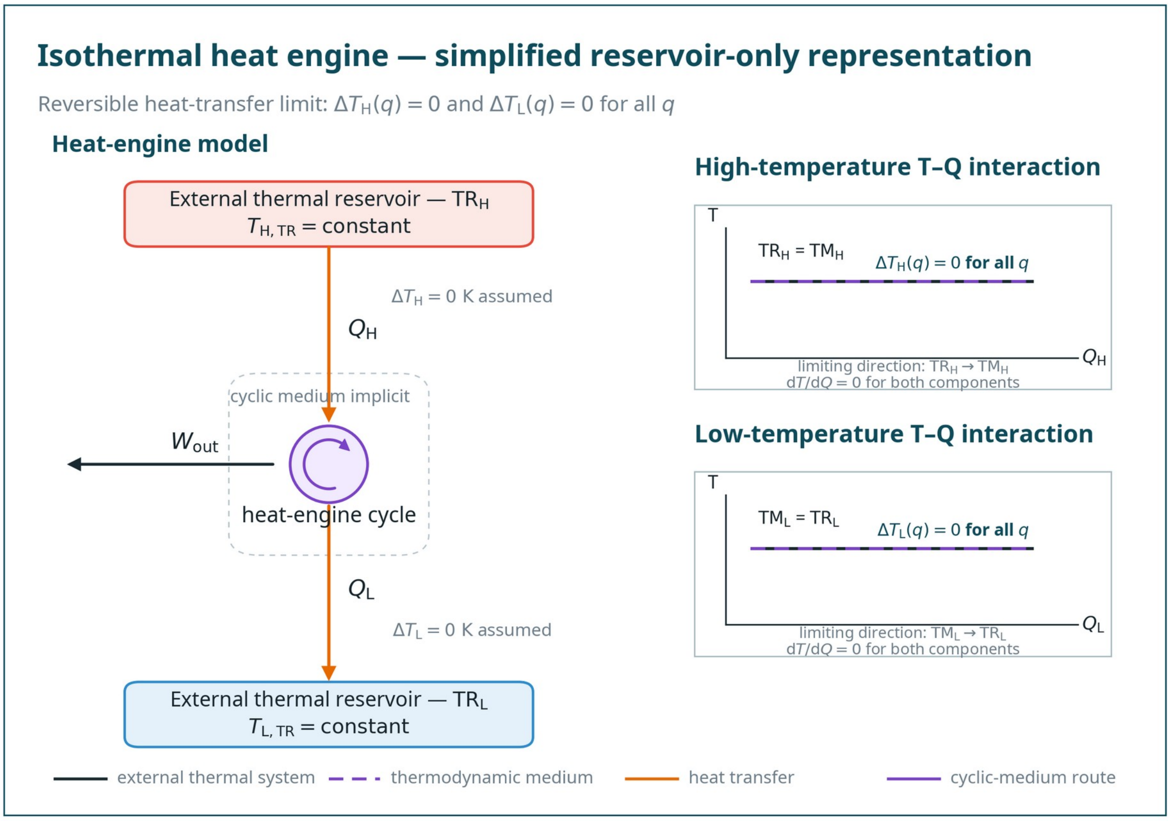

Figure 1. Traditional compact reservoir-only representation of an isothermal heat engine, retaining the familiar reservoir-cycle-reservoir energy-flow organisation on the left and adding paired T-Q diagnostics on the right. QH flows from TRH into the cycle, Wout leaves the cycle and QL is rejected to TRL. Coincident TR/TM trajectories demonstrate ΔTH = ΔTL = 0 K. The A/A architecture is introduced only in Figure 2. Source: author's own elaboration.

TRADITIONAL STARTING POINT The left-hand sketch keeps TMH and TML implicit within the cycle symbol, as in many classical heat-engine schematics. The paired T-Q plots nevertheless expose both medium-side temperatures and their limiting heat-flow directions. The cycle balance is QH = Wout + QL; reversible-limit heat transfer is established only because the paired temperatures coincide pointwise.

3.4 Explicit A/A Heat-Engine Reversible-Limit Architecture

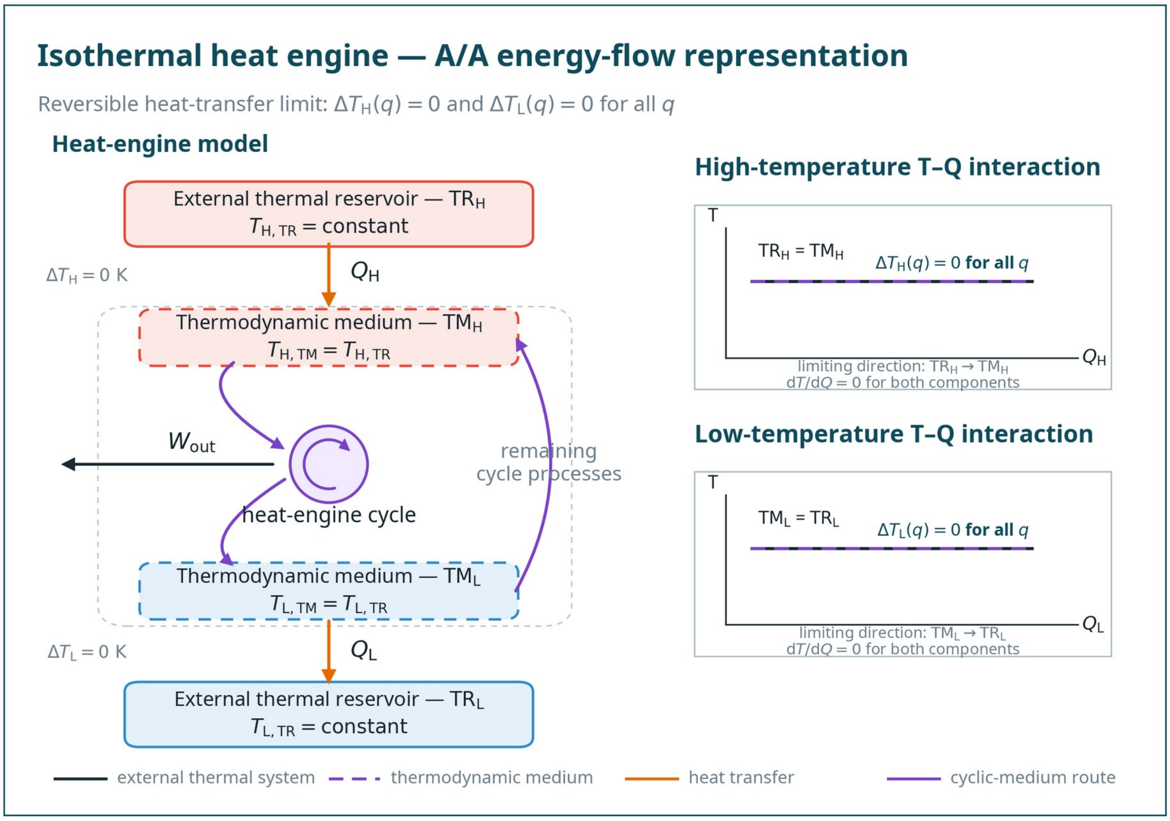

Figure 2. Fully explicit A/A energy-flow representation of the same zero-ΔT isothermal heat-engine case as Figure 1. Separate TMH and TML objects are placed on one cyclic-medium route. This makes temperature ownership and the connection through the remaining cycle processes explicit without changing temperatures, heat directions or cycle function. Source: author's own elaboration.

SAME CASE, GREATER ARCHITECTURAL DISCLOSURE Figures 1 and 2 represent the same heat engine at the reversible heat-transfer limit. Figure 2 expands the previously implicit high- and low-temperature branches of the thermodynamic medium. TMH and TML are therefore two heat-transfer stages of one cyclic medium, not two separate media, and complete-cycle reversibility still requires all remaining cycle processes to be reversible.

3.5 Coincident A/A and B/B Finite-ΔT Heat-Engine Architecture

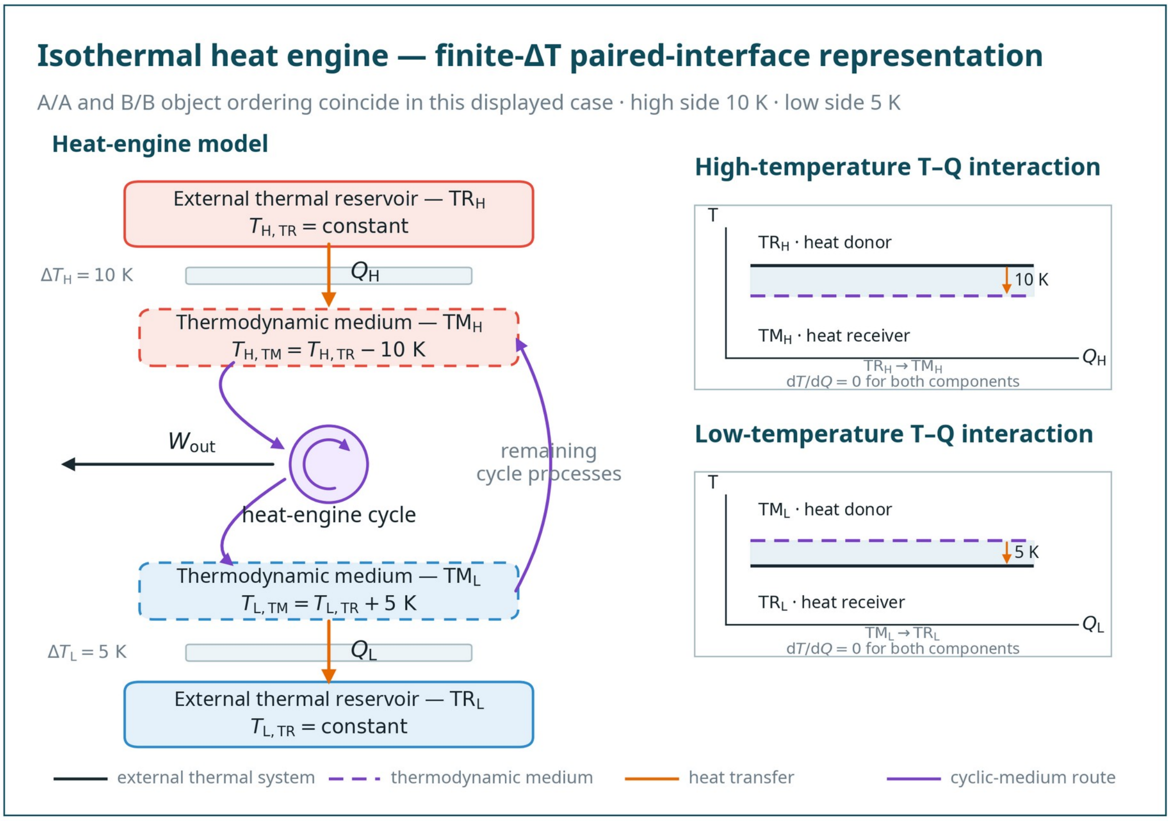

Figure 3. Isothermal heat engine with a 10 K high-side and 5 K low-side temperature difference. TRH is the high-side donor and TMH the receiver; TML is the low-side donor and TRL the receiver. Because donor-to-receiver heat transfer is downward at both interfaces, the explicit A/A energy-flow order and B/B paired-interface order coincide in this special case. The separated horizontal T-Q trajectories have no profile-shape or slope mismatch, yet the finite temperature gaps make both transfers irreversible. Source: author's own elaboration.

A/A = B/B IN THIS DISPLAYED CASE The equality describes an architectural coincidence, not a universal identity. With TRH above TMH and TML above TRL, both A/A and B/B place the same four objects in the same vertical order while preserving QH = Wout + QL. Compared with Figures 1 and 2, Figure 3 also changes the thermal condition from zero to finite gaps. The two interface transfers are irreversible, while complete-cycle reversibility or irreversibility still requires assessment of all remaining cycle processes.

04 | Heat-Pump Paired-Interface Architecture

The paired-interface grammar is now applied to the heat pump. QL enters the thermodynamic medium from the low-temperature external source, QH leaves the thermodynamic medium to the high-temperature external sink, and Win enters the cycle. The sequence moves from the familiar reservoir-only abstraction to explicit A/A and B/B organisations, establishing the isothermal reference architecture used by the three canonical heat-pump cases in Chapters 6–8.

4.1 Traditional Reservoir-Only Heat-Pump Representation

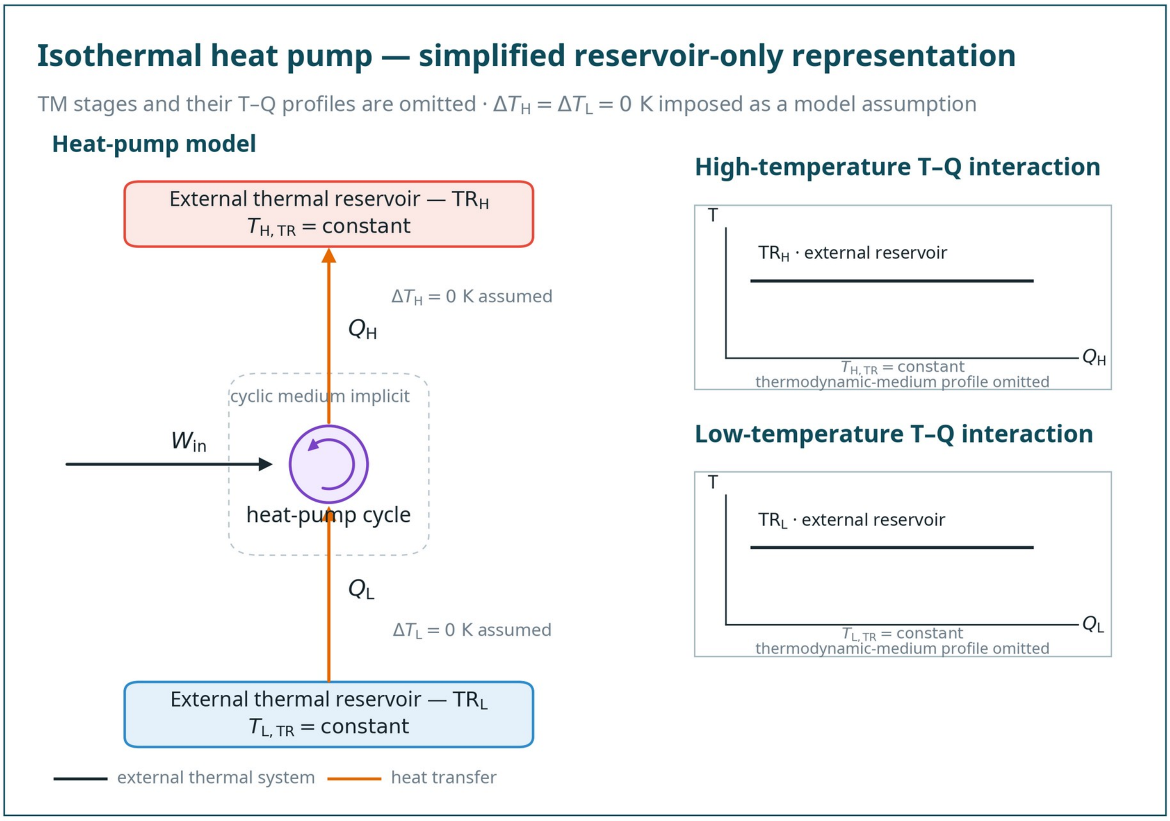

Figure 4. Traditional simplified reservoir-only representation of an isothermal heat pump. The external reservoirs, heat quantities and work input are visible, while the thermodynamic-medium branches and their T-Q trajectories are omitted. The stated ΔTH = ΔTL = 0 K condition is imposed as a model assumption and is not demonstrated by the reservoir-only plots. Source: author's own elaboration.

INTERPRETATION
This compact abstraction identifies the heat-pump function and energy directions. Because the medium-side temperatures are implicit, it cannot independently diagnose the local thermal driving differences. It may represent a reversible heat-transfer limit only when equality of the omitted medium and reservoir temperatures is separately declared.

4.2 A/A Heat-Pump Reversible-Limit Architecture

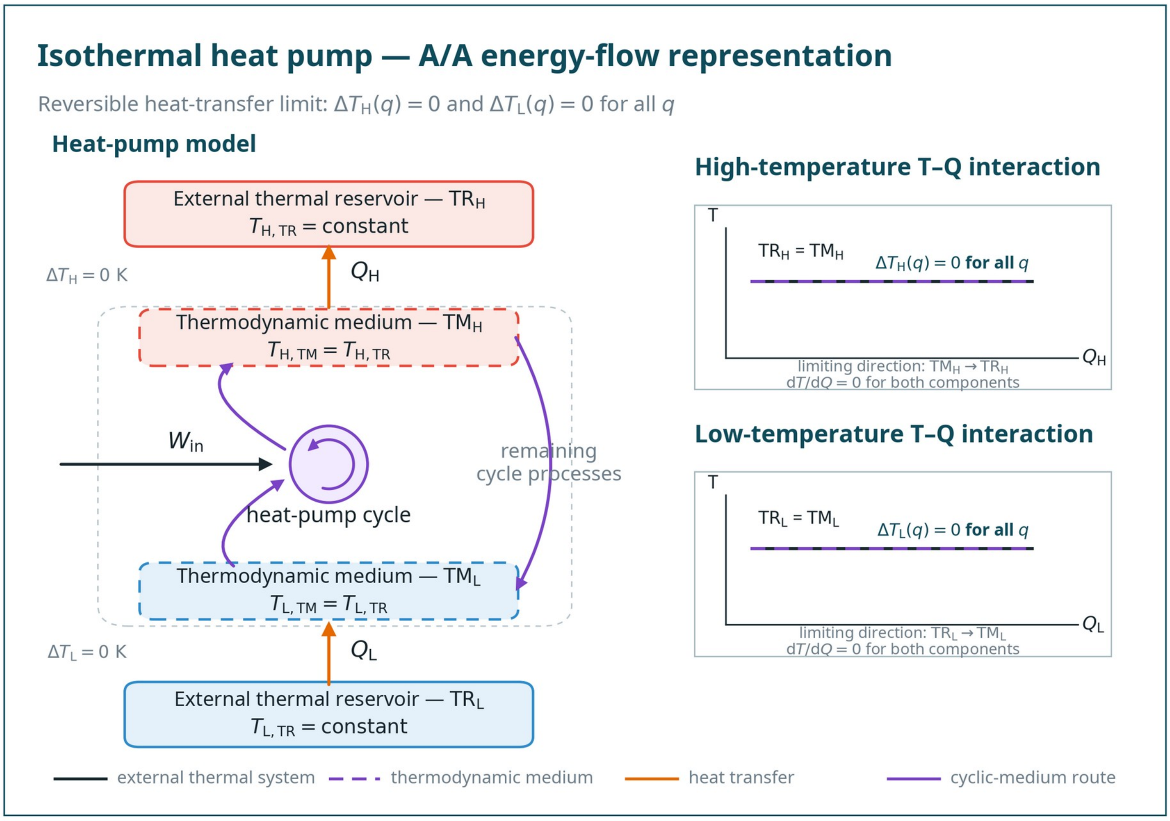

Figure 5. A/A energy-flow architecture for the fully isothermal heat pump at the reversible heat-transfer limit. The thermodynamic medium and external reservoir are separate objects at both thermal sides; their coincident horizontal T-Q trajectories explicitly establish ΔTH = ΔTL = 0 K. Source: author's own elaboration.

REVERSIBILITY BOUNDARY
Pointwise temperature equality establishes reversible-limit heat transfer at the two displayed interfaces. It does not by itself establish that the complete heat-pump cycle is reversible; all remaining internal processes must also satisfy their reversible-limit conditions.

4.3 A/A Heat-Pump Finite-ΔT Architecture

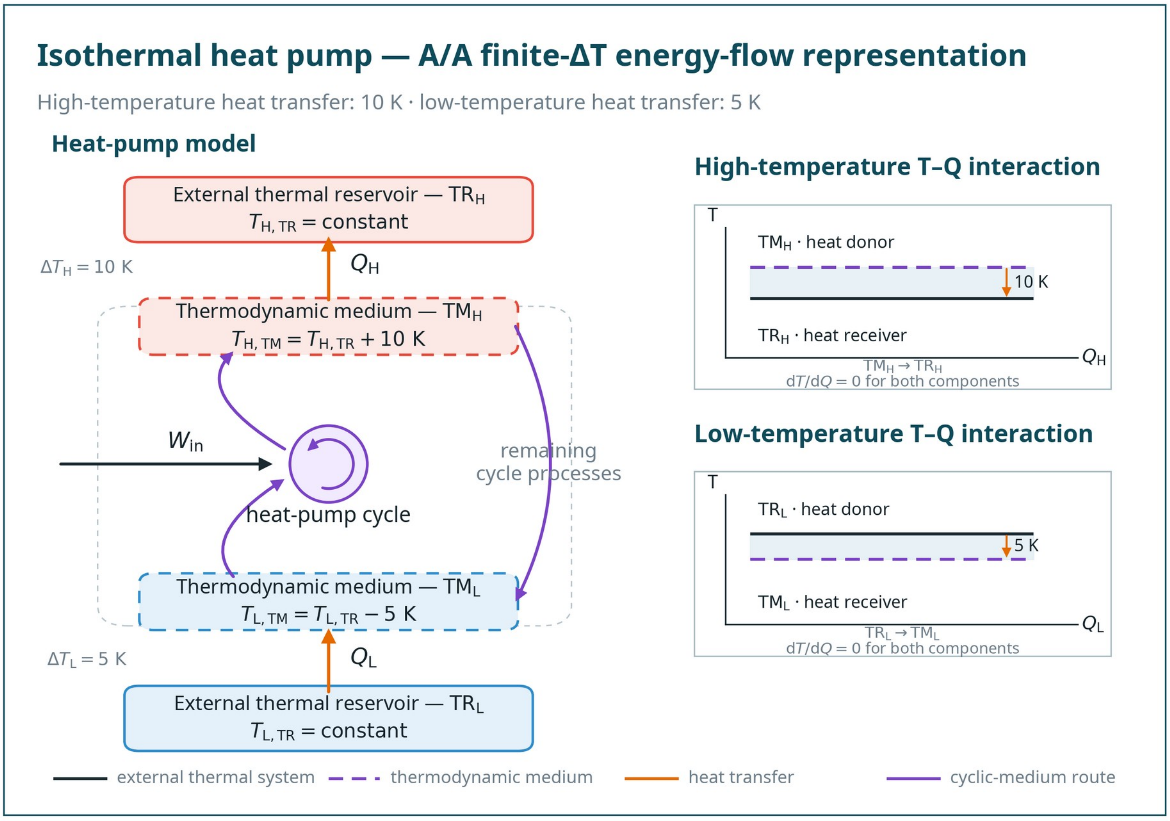

Figure 6. A/A energy-flow architecture for the same fully isothermal heat-pump function with a 10 K high-side and 5 K low-side temperature difference. All four boundary-temperature trajectories remain horizontal, but finite separation makes heat transfer irreversible at both interfaces. Source: author's own elaboration.

ISOTHERMAL IS NOT REVERSIBLE

The thermodynamic medium and reservoirs are isothermal during their respective heat interactions, yet their unequal temperatures create finite-temperature-difference irreversibility. Both paired profiles have zero slope, so no profile-shape mismatch is present. Their finite separation can nevertheless be reduced either by conductance enhancement or by a direction-neutral change to one or both temperature levels that is not attributable solely to increasing UA; these interventions must not be conflated.

4.4 B/B Heat-Pump Finite-ΔT Architecture

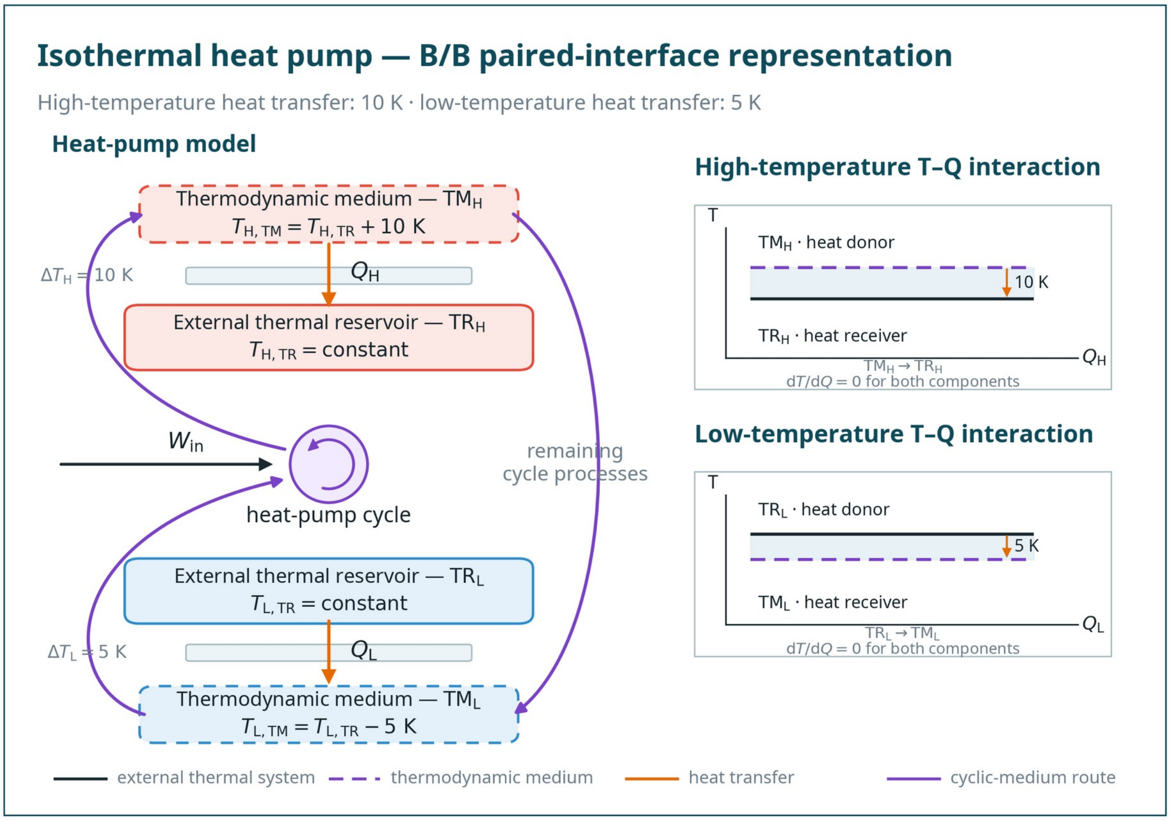

Figure 7. B/B paired-interface architecture applied to the same fully isothermal finite-ΔT heat-pump case as Figure 6. Donor and receiver are positioned directly across each interface and their horizontal T-Q trajectories expose the 10 K high-side and 5 K low-side separations. Source: author's own elaboration.

A/A-B/B READING RULE Figures 6 and 7 do not represent different heat-pump cycles. They preserve the same temperatures, heat directions and cycle function. B/B changes the visual organisation by pairing each heat donor and receiver locally, making temperature ownership, interface direction and the finite gap easier to inspect.

05 | Governing Thermodynamic Conditions

Temperature behaviour, heat-transfer reversibility and complete-cycle reversibility are assessed separately.

5.1 Interaction classification

An interaction is isothermal when both relevant boundary temperatures remain constant throughout that interaction. A finite constant difference does not change that classification. An interaction is non-isothermal when at least one relevant boundary-temperature trajectory varies through the heat-transfer process.

5.2 Reversible local thermal limit

$$T_{\text{donor},j}(q) = T_{\text{receiver},j}(q) \text{ for every } q \in [0,1] \quad (4)$$

Pointwise coincidence is the thermal-interface condition. Equal endpoint temperatures, one zero pinch or equal average temperatures do not establish Equation (4) over the complete interaction.

5.3 Linear trajectory model

$$T(q) = T_{\text{in}} + (T_{\text{out}} - T_{\text{in}})\, q \quad (5)$$

$$\frac{dT}{dQ} = \frac{T_{\text{out}} - T_{\text{in}}}{Q_{\text{tot}}} = \text{constant} \quad (6)$$

The linear model is an idealized representation. A real water or refrigerant trajectory can depart from linearity because heat capacity, phase equilibrium, pressure, composition, heat loss or boundary conditions vary.

5.4 Necessary but insufficient thermal condition

If Equation (4) holds on both heat-transfer branches, those two modelled heat-transfer processes are in their reversible limit. That result does not by itself prove complete-cycle reversibility. Friction, pressure losses, non-reversible compression or expansion, throttling, dissipative mixing, electrical resistance and other mechanisms may remain.

5.5 Four logically distinct statements

Statement	What it establishes	What it does not establish
Both paired trajectories are constant	The interaction is isothermal.	Reversible heat transfer.
Paired trajectories coincide pointwise	Reversible-limit heat transfer for that interface.	Complete-cycle reversibility.
A single terminal gap is zero	A local pinch at that endpoint.	Trajectory coincidence elsewhere.
First-Law balance closes	Energy accounting for the selected cycle boundary.	Admissibility, realizability or reversibility.

5.6 Profile mismatch in the canonical linear cases

For the present three-case set, the visible profile-shape comparison is especially simple. Two horizontal profiles both have $dT/dQ = 0$ and therefore have no slope mismatch, although a finite offset may remain. A horizontal profile paired with a nonzero linear slope has a structural shape mismatch. Thermal Matching may act on temperature level, trajectory shape, heat-duty distribution, or combinations of these. Non-isothermal interactions therefore introduce an additional shape-matching degree of freedom, while a deliberate isothermal temperature-level adjustment may also qualify when the improvement is not attributable solely to conductance enhancement.

06 | Canonical Case I - Fully Isothermal Heat Pump

All four boundary-temperature behaviours are constant; both interfaces retain a finite approach.

6.1 Model definition

$$T_{H,\mathrm{TM}}=60\,°\mathrm{C}, T_{H,\mathrm{ETS}}=50\,°\mathrm{C}, \Delta T_{H}=10\,\mathrm{K} \quad (7)$$

$$T_{L,\mathrm{ETS}}=10\,°\mathrm{C}, T_{L,\mathrm{TM}}=5\,°\mathrm{C}, \Delta T_{L}=5\,\mathrm{K} \quad (8)$$

At the high side, the thermodynamic medium rejects QH to the external sink. At the low side, the external source supplies QL to the thermodynamic medium. Each donor and receiver remains at a constant boundary temperature throughout its respective heat-transfer process.

6.2 Classification

- Heat-pump function: net work input provides the declared high-temperature heating duty.
- Thermal-interaction structure: isothermal under the framework-specific two-interaction classification.
- Heat-transfer reversibility: irreversible at both interfaces because the local differences are 10 K and 5 K.
- Profile-shape mismatch: absent because both members of each pair have dT/dQ = 0; finite-separation heat-transfer irreversibility remains.
- Complete-cycle reversibility: not established; the schematic does not model the internal components.

CONDUCTANCE IMPLICATION

Under a comparison that fixes useful duty while allowing the relevant operating temperature levels to adjust, increasing UA can reduce the required constant approaches. In the ideal limit, both gaps may approach zero. If that reduction results solely from increased conductance, it is conductance enhancement. Because all four trajectories are isothermal in this canonical case, there is no trajectory-shape mismatch; however, a separate deliberate change in one or both temperature levels can still constitute Thermal Matching when it improves local alignment independently of the conductance increase.

6.3 Fully Isothermal B/B Representation

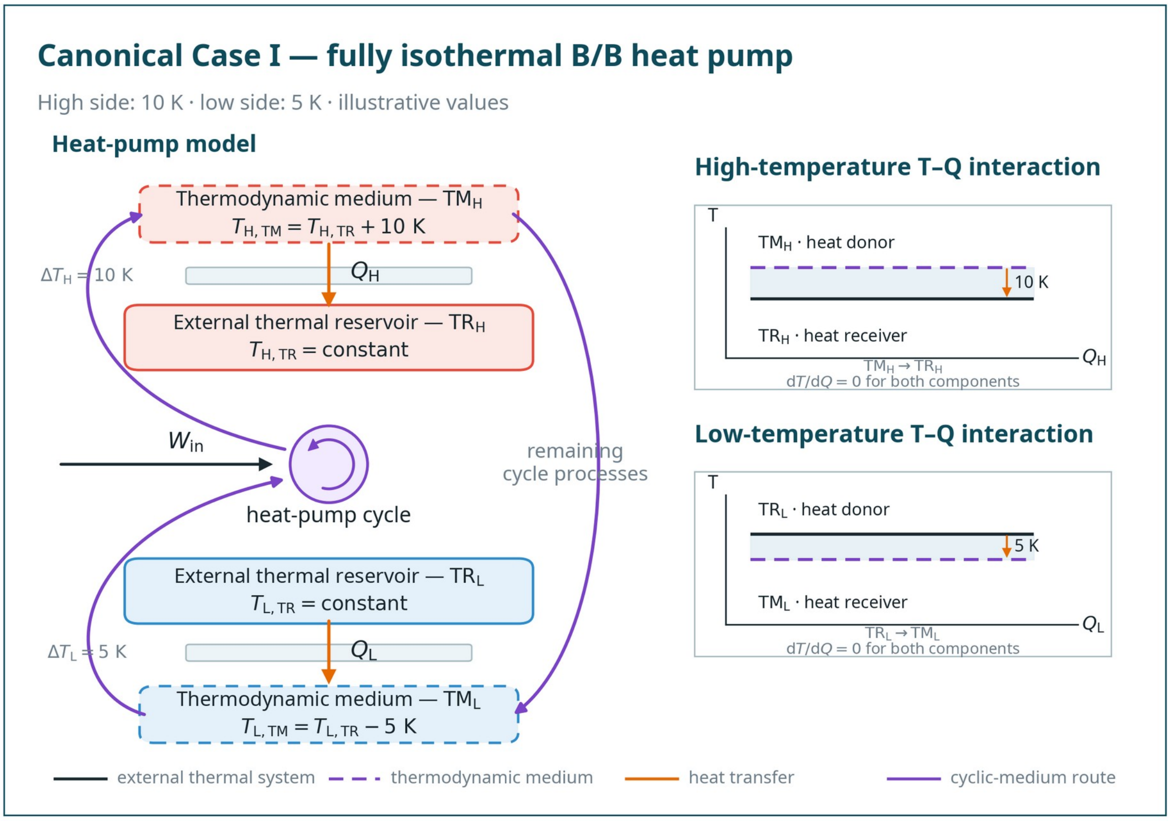

Figure 8. Fully isothermal B/B paired-interface representation. Both paired trajectories are horizontal. Their finite 10 K and 5 K separations classify the two heat-transfer processes as irreversible while the interaction structure remains isothermal. Temperatures are illustrative. Source: author's own elaboration.

READING RULE
The absence of profile-slope mismatch does not mean the heat transfer is reversible. Reversibility requires removal of the constant vertical offset throughout each complete heat-transfer process.

07 | Canonical Case II - External-Source Glide

Only the low-temperature external thermal source changes from isothermal to a linear glide.

7.1 Model definition

$$T_{L,\mathrm{ETS}}(q) = 20\,°\mathrm{C} - 10\,\mathrm{K}\,q, T_{L,\mathrm{TM}} = 5\,°\mathrm{C} \quad (9)$$

$$\Delta T_L(q) = 15\,\mathrm{K} - 10\,\mathrm{K}\,q, \Delta T_{L,min} = 5\,\mathrm{K} \quad (10)$$

The external water-stream example enters the evaporator region at 20 °C and leaves at 10 °C, while the thermodynamic medium remains at 5 °C throughout the modelled low-temperature heat-receiving stage. The external source therefore reaches the same 10 °C terminal level used by the fully isothermal case, but its earlier heat increments are supplied at higher temperatures.

7.2 What remains unchanged

The thermodynamic-medium high- and low-stage temperatures are the same as in Canonical Case I. If the entire internal state path, heat quantities, component performance and all other model conditions are also held fixed, an internal-cycle COP may remain the same. The change in the external source trajectory alone does not prove equality of actual system COP, external pumping work, heat-exchanger performance or total plant performance.

7.3 Classification

- Continuously non-isothermal low-side interaction under the stated linear model.
- Finite and varying local temperature difference: the heat-transfer process is irreversible.
- Trajectory mismatch: external-source slope is nonzero while the thermodynamic-medium slope is zero.
- A 5 K terminal approach does not represent the larger differences present earlier in the interaction.

7.4 External-Source-Glide B/B Representation

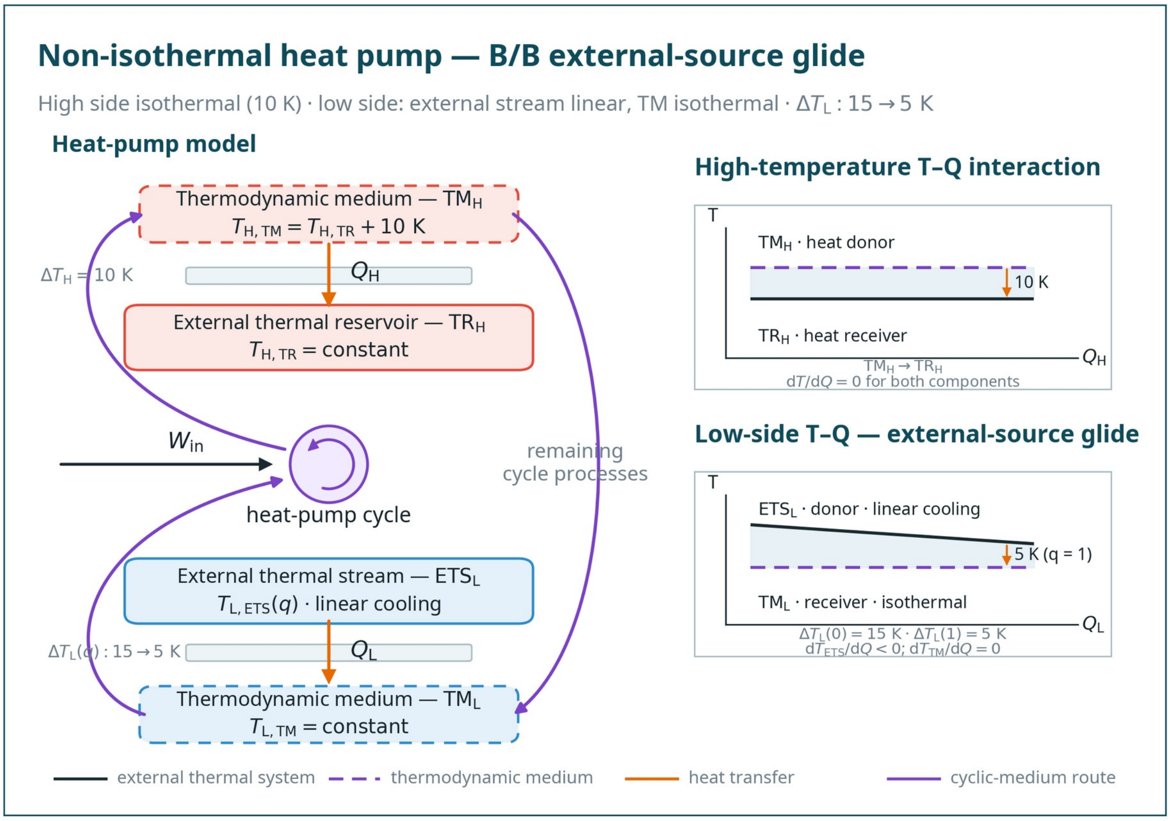

Figure 9. B/B representation with a linearly cooling low-temperature external thermal source and an isothermal thermodynamic medium. The high side is unchanged. The low-side gap decreases from 15 K to 5 K. Source: author's own elaboration.

TERMINOLOGY NOTE
The gliding participant is represented as a finite-capacity external thermal stream rather than an ideal constant-temperature reservoir.

08 | Canonical Case III - Thermodynamic-Medium Glide

The low-temperature external source is isothermal; the thermodynamic medium heats linearly while receiving heat.

8.1 Model definition

$$T_{L,\mathrm{ETS}}=10\,°\mathrm{C}, T_{L,\mathrm{TM}}(q)=-5\,°\mathrm{C}+10\,\mathrm{K}\,q \quad (11)$$

$$\Delta T_L(q)=15\,\mathrm{K}-10\,\mathrm{K}\,q, \Delta T_{L,min}=5\,\mathrm{K} \quad (12)$$

The thermodynamic medium begins the modelled low-temperature heat-receiving stage at -5 °C, well below the 10 °C external source. It rises linearly to 5 °C. Only at the end does it reach the same 5 °C medium temperature used throughout the low-temperature stage of Canonical Cases I and II.

8.2 Consequence for the internal cycle

Unlike Canonical Case II, the thermodynamic-medium trajectory has changed. The internal heat-receiving path, state evolution and possibly the compression/expansion requirements therefore differ. COP must not be assumed unchanged merely because one terminal medium temperature, one minimum approach and the complete high side coincide with the earlier cases.

8.3 Classification

- Continuously non-isothermal low-side interaction under the stated linear medium model.
- Finite and varying local temperature difference: the heat-transfer process is irreversible.
- Trajectory mismatch: the external-source slope is zero while the medium slope is nonzero.
- The 5 K terminal approach is shared with the other cases, but the process-wide temperature history is not.

8.4 Thermodynamic-Medium-Glide B/B Representation

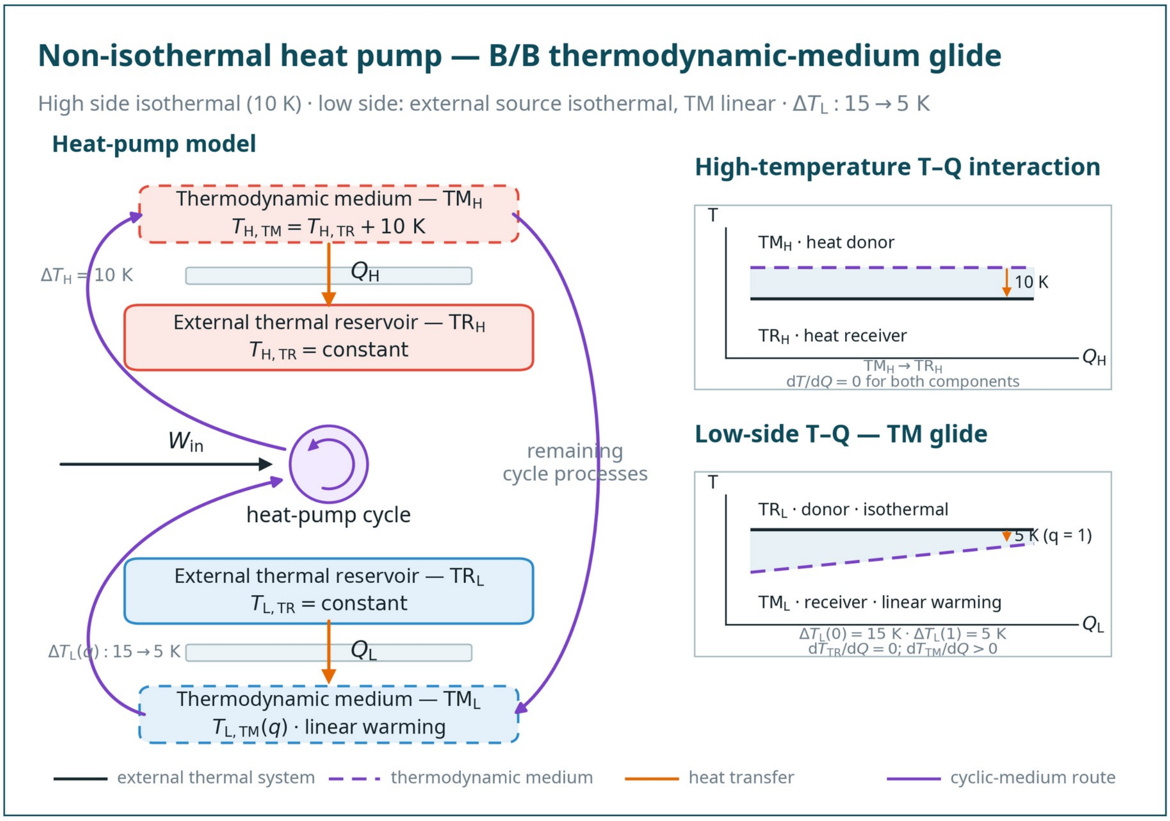

Figure 10. B/B representation with an isothermal low-temperature external source and a linearly warming thermodynamic medium. The low-side gap decreases from 15 K to 5 K. The changed medium trajectory means COP is not assumed unchanged. Source: author's own elaboration.

MODEL BOUNDARY
The linear TM trajectory is a schematic thermodynamic-medium boundary-temperature path. It is not identified with a specific pure or zeotropic refrigerant, pressure profile or phase-equilibrium calculation.

09 | Cross-Case Comparison

The same terminal or minimum temperature approach can coexist with fundamentally different process-wide temperature histories. A single terminal ΔT therefore does not identify whether the external thermal system, the thermodynamic medium, or both are non-isothermal, nor does it establish equivalent cycle behaviour.

Criterion	Fully isothermal	External-source glide	TM glide
High side	Both constant; 10 K gap	Same	Same
Low external system	10 °C constant	20 → 10 °C linear	10 °C constant
Low TM	5 °C constant	5 °C constant	-5 → 5 °C linear
Minimum low-side gap	5 K	5 K at end	5 K at end
Maximum low-side gap	5 K	15 K at start	15 K at start
Low-side profile-shape mismatch	No slope mismatch	Yes	Yes
Low-side heat transfer	Irreversible	Irreversible	Irreversible
Internal TM path vs Case I	Reference path	Unchanged by definition	Changed
COP equality vs Case I	Reference	Conditional only	Not assumed
Can UA alone remove retained profile-shape mismatch?	No retained profile-shape mismatch	No, under fixed one-flat/one-glide model	No, under fixed one-flat/one-glide model
Process-wide low-side temperature history	constant–constant	glide–constant	constant–glide
What the shared 5 K minimum/terminal gap does not show	5 K gap throughout	earlier gap up to 15 K; ETS path changes	earlier gap up to 15 K; TM path changes

COMPARISON PRINCIPLE

A comparison is valid only after the useful duty, system boundary, heat quantities, terminal conditions, area, flow conditions, component model and adjustable variables have been declared. Equal endpoints do not make two process trajectories thermodynamically equivalent. Equal terminal approaches do not imply equal local ΔT(Q) distributions, equal trajectory shapes, equal thermodynamic-medium state paths, or equal cycle-level work consequences.

10 | Conductance Enhancement and Thermal Matching

An irreversible heat engine or heat pump can contain several distinct irreversible processes. This chapter isolates the external heat-transfer contribution by examining the paired donor/receiver temperature trajectories and distinguishes conductance enhancement from changes in trajectory compatibility.

10.1 Overall Heat-Transfer Coefficient and Conductance

The overall heat-transfer coefficient U is an intensive coefficient. The product UA is the heat-transfer conductance and includes the heat-transfer area. For a declared steady two-stream approximation, the familiar relation Q = UA ΔTlm may be useful, but it does not determine which temperatures, flows or duties change when U or A is changed.

$$\dot{Q} = U\,A\,\Delta T_{\mathrm{lm}} \quad (13)$$

A statement that higher U or UA reduces temperature differences is therefore conditional. It is meaningful when useful heat duty is fixed while the control or design space permits one or more temperature levels, mass flows, areas or terminal states to adjust. In the fully isothermal case, both profiles are horizontal and therefore already shape-matched. Reducing their parallel offset solely by increasing conductance is conductance enhancement, whereas a separate deliberate temperature-level adjustment may constitute Thermal Matching. Figures 11 and 12 isolate what conductance enhancement can and cannot change. Figures 13-14 then illustrate trajectory-shape adaptation for a non-isothermal paired-interface case. Figure 15 then combines both interfaces in an illustrative dual-glide before-and-after engineering synthesis. If Q and all inlet and outlet temperatures are fixed, the plotted trajectories do not automatically move when UA is increased.

10.2 Conductance Enhancement of Profile-Matched Isothermal Trajectories

Figure 11 isolates conductance enhancement for two already profile-matched isothermal trajectories. At fixed heat duty, and with an operating temperature level permitted to adjust, increasing UA reduces the constant temperature separation required to transfer that duty. Both trajectories remain horizontal throughout: their shapes and slopes do not change.

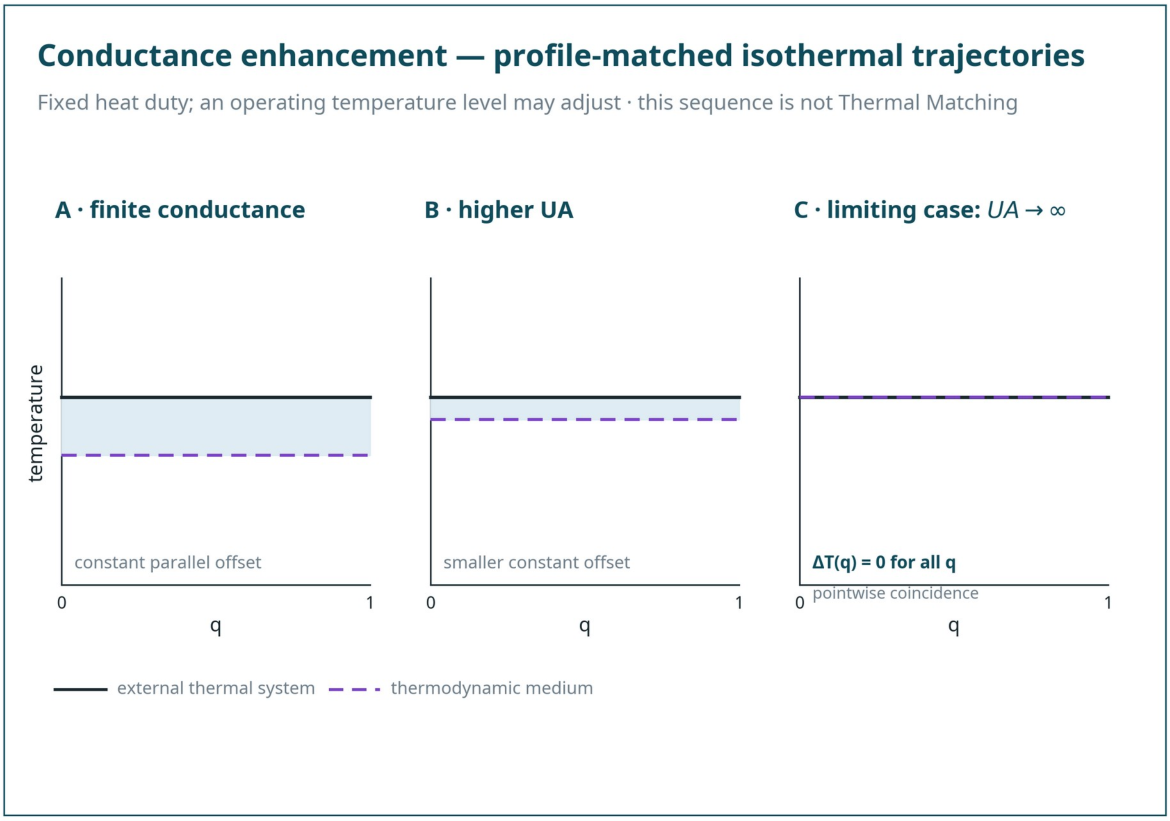

Figure 11. Conductance enhancement of two already profile-matched isothermal trajectories. At finite conductance the profiles have a constant parallel separation; at higher UA the separation is smaller. In the theoretical limit UA tends to infinity, ΔT(q) tends to zero for every q and the profiles coincide pointwise. This sequence is conductance enhancement, not Thermal Matching, because the temperature-gap reduction is attributed solely to increasing UA; no independent trajectory-level, shape or heat-duty-distribution adaptation is imposed. The operating-temperature level is assumed able to adjust; if duty and all terminal temperatures are fixed, increasing UA does not by itself move the plotted profiles. Source: author's own elaboration.

CONDUCTANCE ENHANCEMENT — NOT THERMAL MATCHING
In the sequence shown, the paired trajectories are isothermal and already profile-matched in every panel. Their decreasing parallel offset results solely from increasing conductance, so this sequence is conductance enhancement rather than Thermal Matching. The UA → ∞ endpoint reaches the reversible heat-transfer limit, but arriving at that limit through conductance enhancement does not reclassify the intervention as Thermal Matching. Infinite UA is a theoretical limit, not a realizable finite design value.

10.3 Thermal Matching: Direction-Neutral Definition

Thermal Matching is the proposed framework-specific engineering method for characterising and improving the local alignment of the thermodynamic-medium and external-system boundary-temperature trajectories at one or both heat-transfer branches. It may act on trajectory shape, temperature level, heat-duty distribution, or combinations of these, provided that the resulting configuration reduces the relevant local temperature differences under a stated comparison basis without relying solely on conductance enhancement. The method is therefore not restricted to non-isothermal interactions. Non-isothermal cases add trajectory-shape matching as an additional design degree of freedom. Candidate interventions include medium selection, mixture composition, pressure levels, mass flows, heat-capacity rates, phase-change behaviour, flow arrangement, staging and heat-duty distribution.

SCOPE OF THERMAL MATCHING
Thermal Matching concerns the local compatibility of the paired boundary-temperature trajectories of the thermodynamic medium and the external thermal system. It is distinct from conductance enhancement: increasing U and/or A changes the heat-transfer conductance UA and addresses resistance to heat transfer, whereas Thermal Matching addresses compatibility of the temperature trajectories and levels. Both may be applied together. A finite UA value is not itself an irreversibility measure; for finite heat duty, heat transfer across a finite local boundary-temperature difference is irreversible. This report uses the paired T-Q trajectories and local or reported minimum, maximum and terminal temperature differences to diagnose the location, distribution and change of thermal mismatch. These interface quantities are not used here as a separate scalar measure of the thermodynamic magnitude of irreversibility. Thermal Matching does not address compressor or expander inefficiency, throttling, pressure loss, friction, mixing, leakage, electrical losses or other internal cycle irreversibilities.

10.4 Operational Assessment of Thermal Matching

Thermal Matching is assessed against a declared comparison basis. The complete paired temperature trajectories matter: one terminal approach, one pinch point or a slope comparison alone is insufficient when the local gap varies through the transferred-heat coordinate.

A compact assessment should therefore state: (1) what function, duty, boundary and terminal conditions are held fixed; (2) which physical variable or trajectory is changed; (3) the before-and-after paired T-Q trajectories; (4) the resulting local, minimum, maximum and terminal temperature differences; and (5) any COP or work-input consequence only when the cycle model is sufficiently specified.

ASSESSMENT RULE
A Thermal-Matching change is favourable within the stated comparison basis when better alignment of trajectory shape, temperature level or heat-duty distribution reduces the relevant local temperature differences without creating an inadmissible temperature crossover and without the improvement being attributable solely to conductance enhancement. At finite conductance, equal dT/dQ slopes are neither a necessary nor a sufficient condition for the thermodynamic optimum. Figures 13 and 14 are illustrative slope-adjustment examples only and do not prescribe which trajectory must move or what the finite-conductance optimum must look like.

RELATION TO THE FORMAL CORE
Paired-interface modelling makes Thermal Matching and Thermal Mismatching visible at the heat-transfer interfaces. The Formal Core provides the broader cycle-to-cycle method: the complete irreversible cycle is compared with its corresponding reversible cycle, with benchmark-relative lost work for a heat engine or excess work input for a heat pump or refrigerator used to quantify the complete-cycle work consequence of irreversibility. The local T-Q geometry and finite temperature differences diagnose the thermal interfaces; they do not by themselves provide the cycle-level work penalty. If the work consequence of one or both finite-temperature-difference heat-transfer processes is to be isolated, all other irreversible mechanisms must be held fixed or excluded by the declared comparison class. Otherwise the lost or excess work represents the combined cycle-level consequence of all irreversible mechanisms that differ between the compared cycles. This companion does not repeat the Formal Core derivation.

10.5 Conductance Enhancement with One-Sided Glide

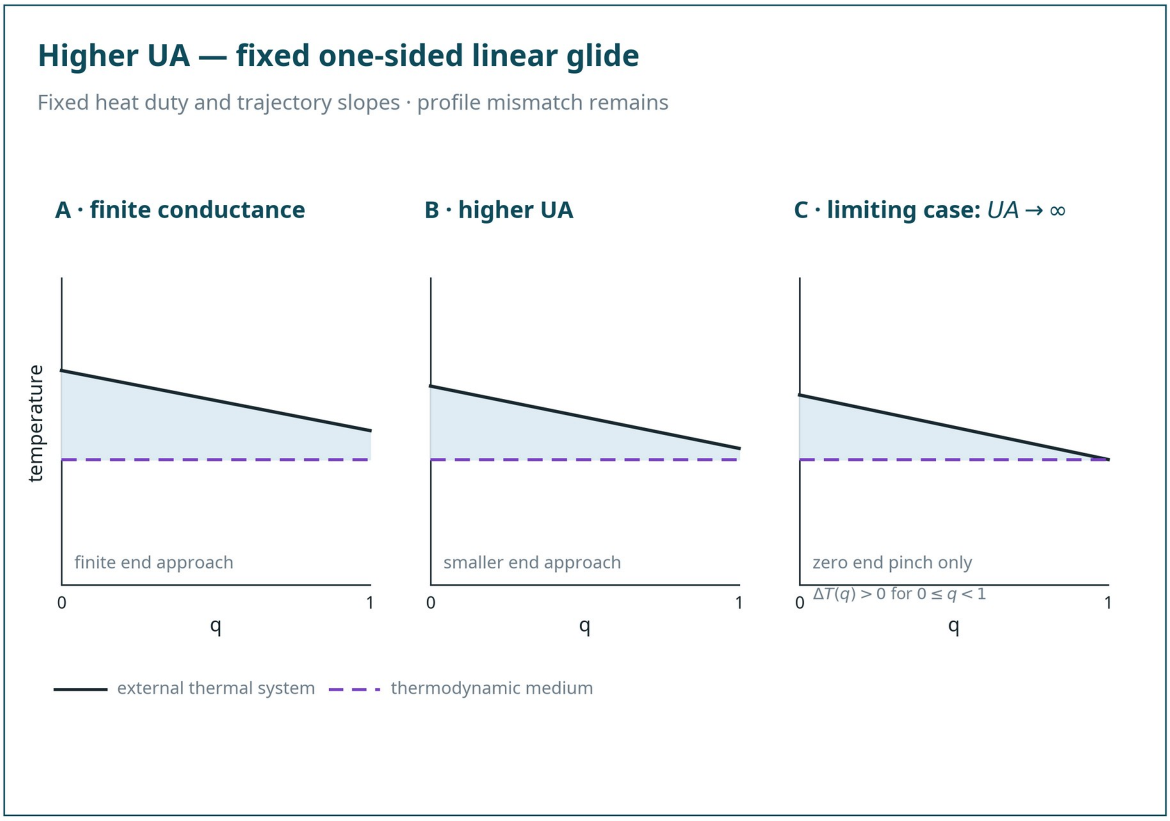

Figure 12. Conductance enhancement applied to a fixed one-sided linear temperature glide. At finite and higher UA, the terminal approach decreases while the slope mismatch is unchanged. In the limiting case UA tends to infinity, a zero end pinch occurs at q = 1, but a finite process-wide gap remains for 0 <= q < 1. Increasing conductance alone therefore cannot remove the glide mismatch. Source: author's own elaboration.

The figure isolates conductance enhancement: heat duty and the one-sided linear glide are fixed, while the operating temperature level may adjust. The narrowing shaded area represents a smaller local temperature separation, not a change in trajectory compatibility.

CONDUCTANCE LIMIT
When unequal slopes are retained, even the ideal UA-to-infinity limit can create only a terminal zero pinch. It cannot make the two temperature trajectories coincide over the complete heat-transfer duty.

10.6 Progressive Thermal Matching - Vanishing-Separation Limit

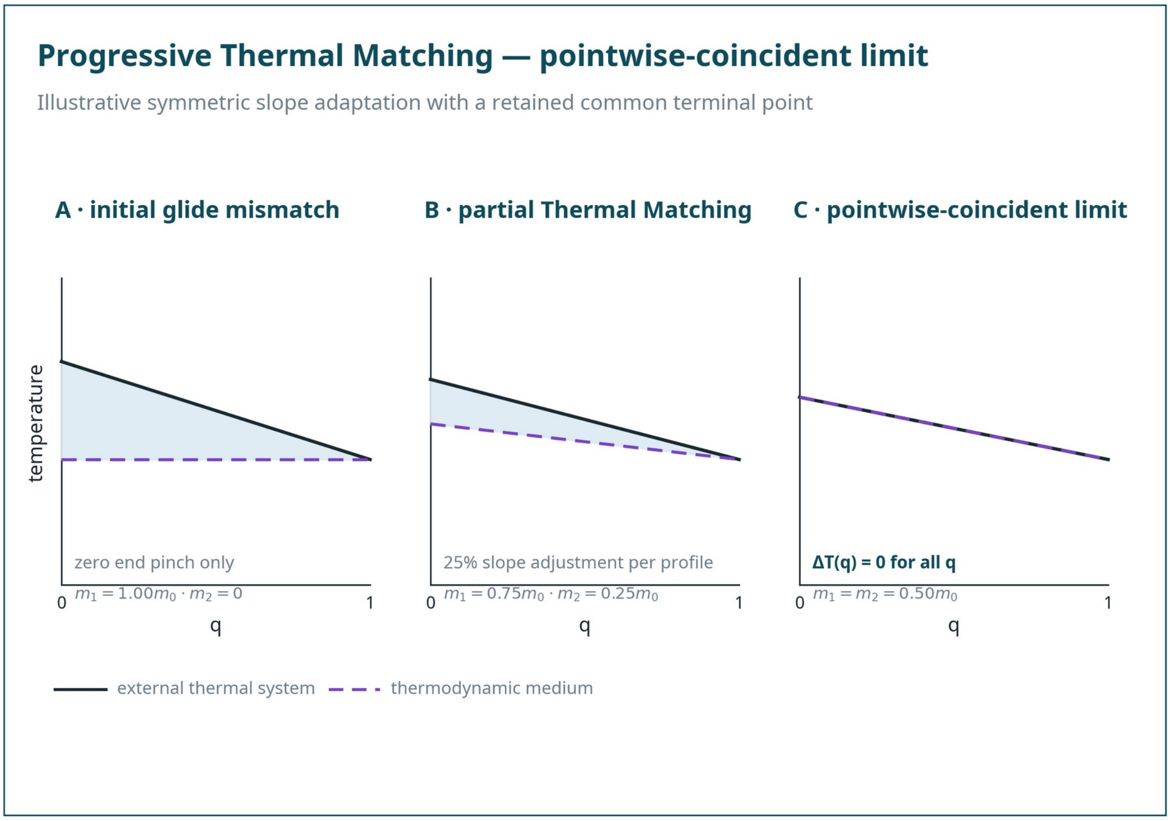

Figure 13. One illustrative route for progressive Thermal Matching with a retained common terminal point. For visual symmetry, both trajectories are modified: in the middle panel, each slope is shifted by 25% of the initial slope difference, reducing the slope mismatch by 50%. In the final panel both slopes equal 0.50 times the initial glide and the profiles coincide pointwise. This is the vanishing-local-separation heat-transfer limit; the symmetric adjustment is not a definitional requirement. Source: author's own elaboration.

For the linear trajectories shown, equal slopes remove the slope mismatch, but they are not sufficient for reversible heat transfer. Pointwise coincidence occurs here because the equal-slope profiles also share the same terminal point, so their local temperature separation vanishes throughout the duty.

PROFILE MATCHING AND REVERSIBLE LIMIT
In this constructed limiting sequence, both trajectories are modified and the common endpoint is retained. The slope mismatch and remaining offset then vanish together, producing pointwise coincidence. Thermal Matching itself does not prescribe symmetric movement or require either the external-system or medium trajectory to be the fixed reference.

10.7 Progressive Thermal Matching at Finite UA

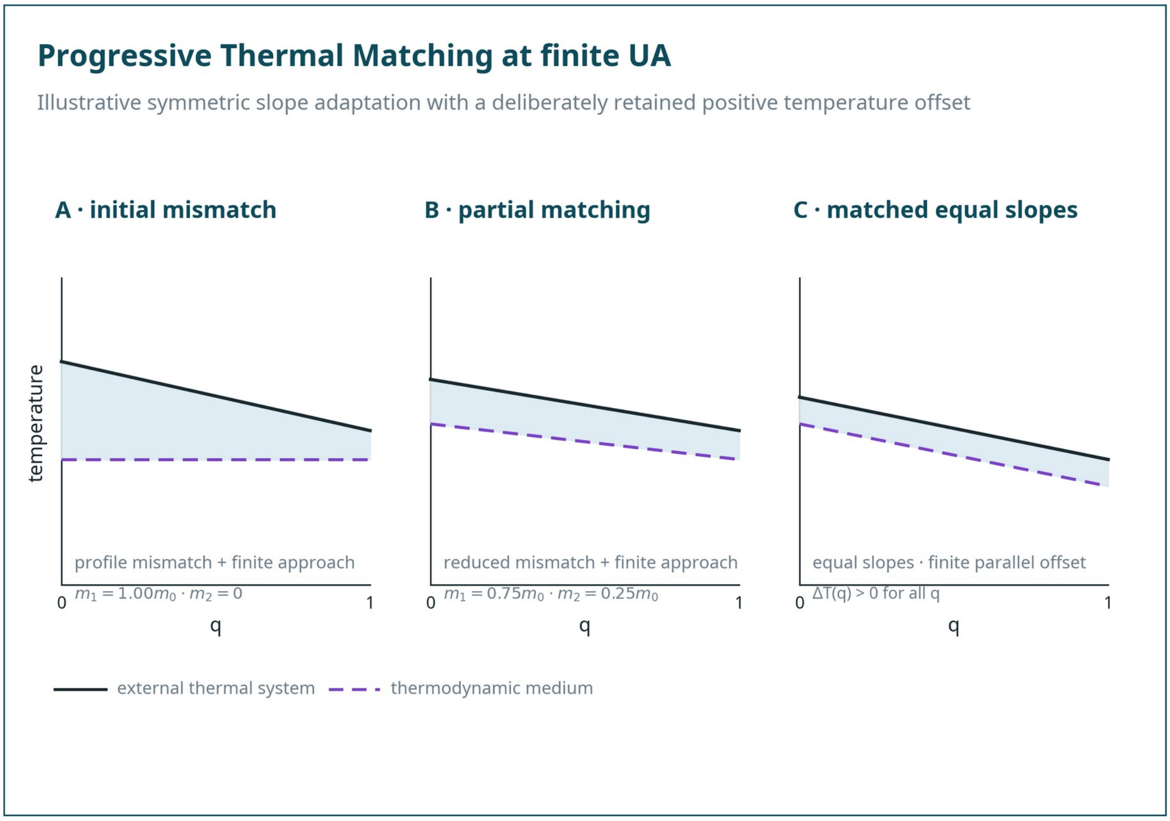

Figure 14. The same illustrative two-trajectory slope-adaptation sequence at finite UA. The upper trajectory remains visibly displaced above the lower trajectory. In the final panel the slopes are equal and the profile-shape mismatch is removed, but a finite parallel temperature gap remains. Heat transfer therefore remains thermodynamically irreversible. The retained offset and equal-slope endpoint are conditions of this example, not a general finite-conductance optimum criterion. Source: author's own elaboration.

This sequence separates the illustrated slope adaptation from the remaining heat-transfer approach. In a non-isothermal paired-interface case, equal slopes can remove one form of profile-shape mismatch while finite conductance may still require a positive local driving temperature difference for finite heat duty. Equal slopes do not define the general finite-conductance optimum: depending on the declared constraints and adjustable variables, an optimum may retain non-zero, non-uniform local temperature differences and unequal slopes. More generally, Thermal Matching may change trajectory shapes, levels or heat-duty distribution; this figure intentionally retains a parallel offset to isolate one finite-UA example.

THERMAL MATCHING BOUNDARY
The displayed intervention leaves a finite parallel offset because that offset is intentionally held fixed; this is a property of the example, not a general restriction on Thermal Matching. A different intervention may change trajectory levels, shapes or heat-duty distribution, provided that the improvement is not attributable solely to conductance enhancement. Isothermal temperature-level alignment may also qualify as Thermal Matching under that same distinction. No such intervention remedies irreversibilities elsewhere in the heat-engine or heat-pump cycle.

10.8 Engineering Synthesis — Dual-Glide Thermal Matching at Finite Conductance

The preceding cases isolate individual thermal-interaction features. This synthesis combines them in one heat-pump representation in which both the thermodynamic medium and the external thermal system follow linear non-isothermal trajectories at both the high- and low-temperature interfaces. The comparison is intentionally qualitative and contains no equipment-performance or optimum calculation.

Configuration A represents the initial dual-glide mismatch. Configuration B retains both external-system trajectories while adapting the thermodynamic-medium trajectories so that the absolute TM–ETS slope mismatch is reduced by 50% at both interfaces. The relevant local temperature differences are also reduced while remaining positive. Configuration B therefore represents improved Thermal Matching at finite conductance; it is not identified as the thermodynamic optimum or as the reversible limit.

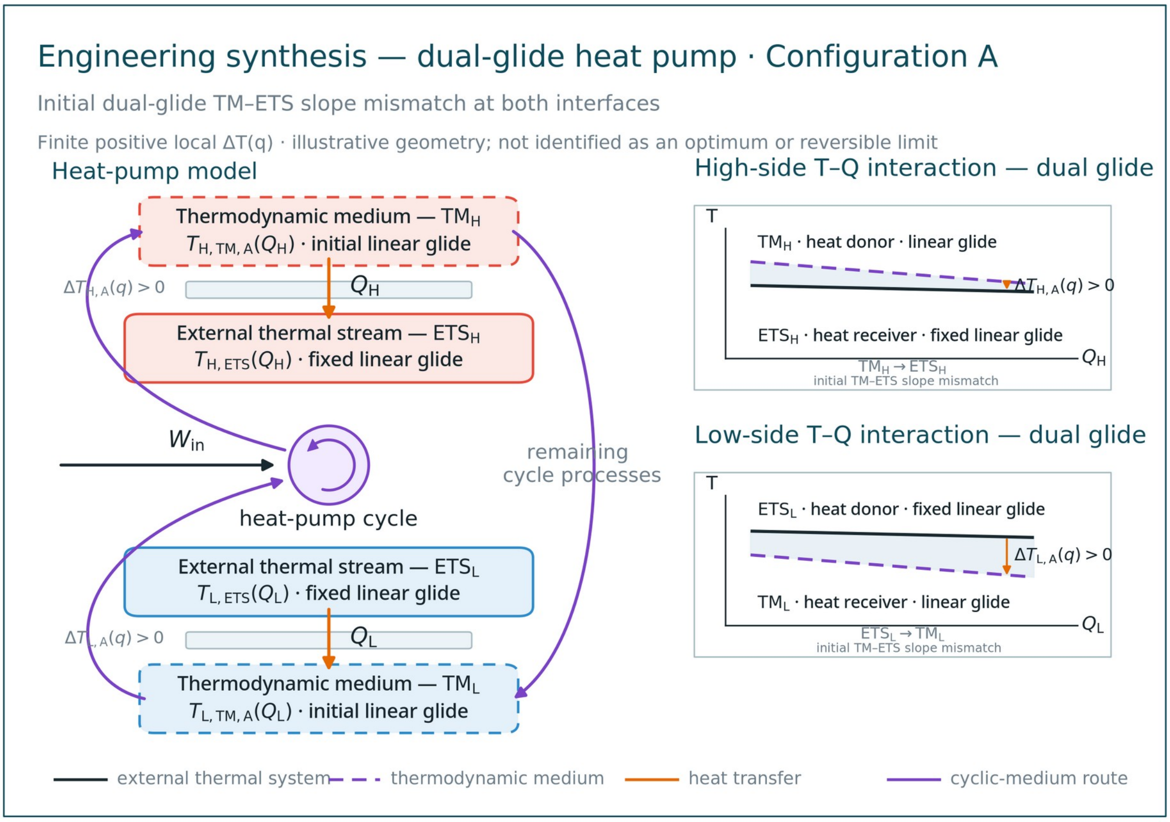

Figure 15a. Engineering synthesis — Configuration A. Both participants follow linear non-isothermal T–Q trajectories at the high- and low-temperature interfaces. The displayed TM–ETS slope mismatch and finite positive local temperature differences define the initial dual-glide condition. Geometry is illustrative; no equipment-performance or optimum claim is made. Source: author's own elaboration.

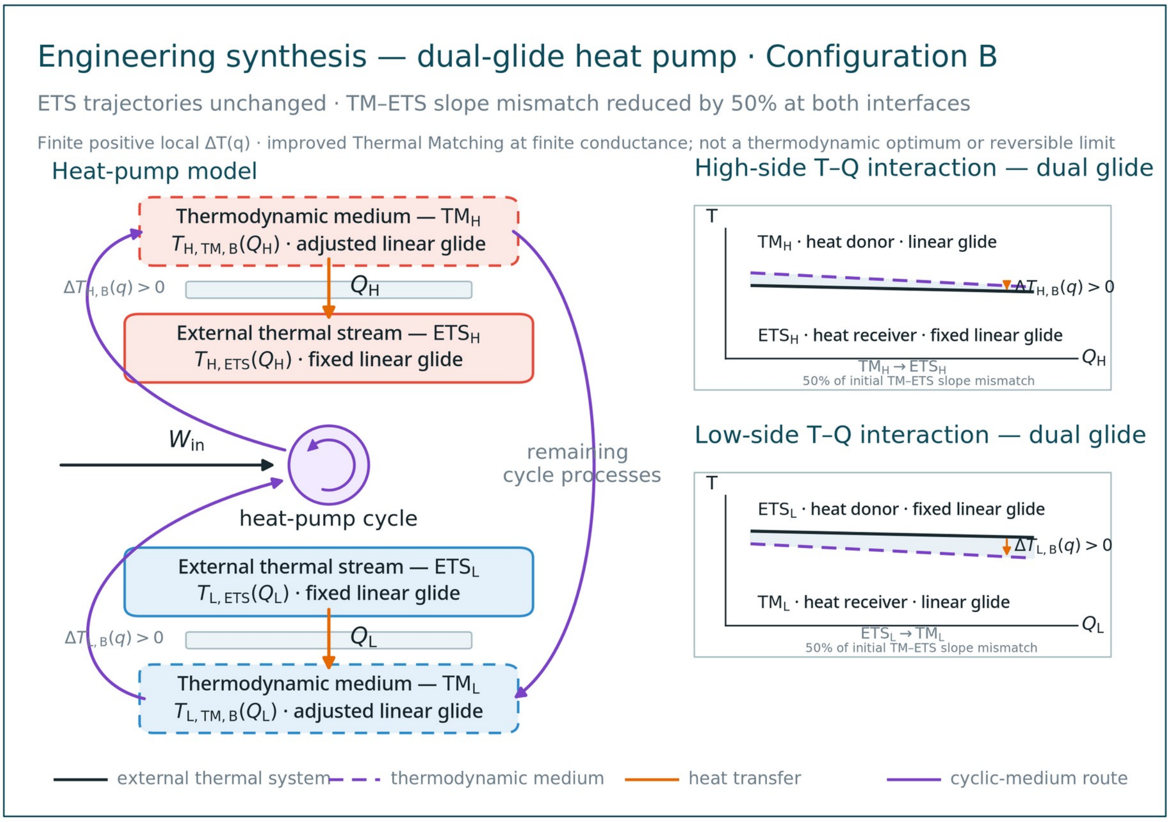

Figure 15b. Engineering synthesis — Configuration B. The external-system trajectories are retained while the thermodynamic-medium trajectories are adapted so that the absolute TM–ETS slope mismatch is reduced by 50% at both interfaces. The local temperature differences are reduced but remain positive. The figure illustrates improved Thermal Matching at finite conductance, not a thermodynamic optimum or the reversible limit. Source: author's own elaboration.

ENGINEERING INTERPRETATION

This before-and-after synthesis shows that Thermal Matching can act simultaneously at both external heat-transfer interfaces. Improved matching does not require equal slopes and, at finite conductance, may coexist with unequal slopes, non-uniform local temperature differences and thermodynamically irreversible heat transfer. The preferred finite-conductance configuration depends on the declared cycle function, useful duty, boundary conditions, conductance and other engineering constraints. In the reversible local limit the paired trajectories would coincide pointwise; that limiting condition is not shown here.

11 | Quantification Levels, COP and Work-Input Implications

Performance belongs to a complete declared cycle and comparison class, not to a T-Q sketch alone.

$$\mathrm{COP}_H = \frac{Q_H}{W_{\mathrm{in}}}, \mathrm{COP}_L = \frac{Q_L}{W_{\mathrm{in}}} \quad (14)$$

A B/B figure supplies essential heat-temperature information but not a complete cycle model. A numerical COP additionally requires heat quantities, internal state relations, component behaviour, pressure levels, mass flows, closure and the selected useful-duty definition.

11.1 Two Levels of Quantification

This companion distinguishes two complementary levels of quantification. The first characterises Thermal Matching or Thermal Mismatching directly at the paired heat-transfer interfaces. The second quantifies the thermodynamic work consequence at complete-cycle level. The two levels are physically related but are not interchangeable.

Level	What is quantified	Primary representation / output
Interface level	The distribution and change of Thermal Matching or Thermal Mismatching at an individual high- or low-temperature heat-transfer interface.	Paired T–Q trajectories; local ΔT(Q); minimum, maximum and terminal approaches; trajectory shape and slope differences; crossover check. These are interface diagnostics, not a scalar thermodynamic magnitude of irreversibility.
Cycle level	The complete-cycle work consequence of irreversibility relative to the corresponding reversible cycle under the declared function, boundary, useful duty and comparison conditions.	Benchmark-relative lost work for a heat engine or excess work input for a heat pump or refrigerator within the Formal Core method. This aggregate quantity does not by itself allocate the penalty to one heat-transfer interface or other local mechanism.

At interface level, the paired T-Q trajectories, local ΔT(Q), terminal approaches and trajectory-shape or slope differences describe where thermal mismatch occurs, how it is distributed over the transferred-heat coordinate, and how it changes between configurations. They do not constitute a standalone scalar quantification of the thermodynamic magnitude of irreversibility produced by that individual heat-transfer interface. At cycle level, the complete irreversible cycle is compared with its corresponding reversible cycle. That comparison yields the aggregate benchmark-relative work consequence for the declared comparison class.

QUANTIFICATION BOUNDARY
Finite boundary-temperature differences at one or both external heat-transfer interfaces establish heat-transfer irreversibility and are retained as local diagnostic information. In this companion, their thermodynamic magnitude is not quantified by assigning a separate lost-work value to an individual interface. The work consequence is quantified from the complete-cycle comparison between the irreversible cycle and its corresponding reversible cycle. Attribution of a change in lost or excess work to one or both finite-temperature-difference heat-transfer processes is valid only when the comparison class holds all other irreversible mechanisms fixed or excludes them. If other mechanisms change simultaneously, the resulting lost or excess work is their combined cycle-level consequence.

11.2 Conditional Comparison Statements

- Fully isothermal case: reducing both external approach temperatures can reduce required temperature lift and work input when the cycle and control model permit the medium levels to move accordingly.
- External-source-glide case: the internal thermodynamic-medium trajectory may remain identical to the fully isothermal case, but equality of system COP is permitted only when QH, QL, Win and all other relevant internal and auxiliary conditions are also identical.
- Thermodynamic-medium-glide case: the internal heat-receiving path changes; COP is therefore recalculated rather than inherited from either previous case.
- For every case, improving only the two modelled thermal interfaces does not remove compressor, expansion, frictional, pressure-drop or auxiliary-work penalties.

11.3 Relation to the Corresponding Reversible Cycle

This is the second quantification level. Within the broader Formal Core method, the complete irreversible cycle is compared with its corresponding reversible cycle under the same declared function, system boundary, useful duty and comparison conditions. The associated cycle-level work consequence is expressed as benchmark-relative lost work for a heat engine or excess work input for a heat pump or refrigerator. This quantity is aggregate at the selected cycle boundary: it does not automatically allocate the total to a particular heat-transfer interface, compressor, pressure loss or other irreversible mechanism. To attribute a change specifically to one or both finite-temperature-difference heat-transfer processes, the comparison class must hold the remaining irreversible mechanisms fixed or exclude them. This technical companion does not reproduce the formal cycle construction or calculation.

12 | Generalized Configuration and Reporting Protocol

A compact reusable protocol for future paired-interface Thermal Matching cases.

1. Declare the cycle function: heat pump for useful high-temperature duty or refrigerator for useful low-temperature duty.
2. Define the cyclic-system boundary and identify the single thermodynamic medium and all external thermal systems.
3. Classify the high and low interactions independently as isothermal, piecewise-isothermal, continuously non-isothermal or mixed.
4. Assign donor and receiver roles and define QH, QL, Win and the sign convention.
5. Select a common local Q or normalized q coordinate for each paired interface.
6. Specify both boundary-temperature trajectories and state whether each is measured, calculated, assumed or illustrative.
7. Plot both trajectories and report local, minimum, maximum and terminal temperature differences where relevant.
8. Classify heat-transfer irreversibility separately from complete-cycle reversibility.
9. Distinguish conductance enhancement from Thermal Matching; within Thermal Matching distinguish temperature-level, trajectory-shape and heat-duty-distribution changes where relevant.
10. State the comparison class before drawing COP, work-input or optimisation conclusions. If a change in cycle-level lost or excess work is attributed to one or both external heat-transfer processes, state explicitly which other irreversible mechanisms are held fixed or excluded.
11. Verify First-Law closure, constitutive sufficiency, process/cycle closure, admissibility and architecture claims.
12. Report epistemic status, source basis, limitations and validation status.

MINIMUM FUTURE FIGURE SET
Every new canonical case should include one B/B architecture diagram, one high-side T-Q panel, one low-side T-Q panel, one compact assumption block and one status sentence covering interaction structure, finite-temperature-difference irreversibility, the relevant Thermal-Matching degree of freedom, and COP qualification.

13 | What Thermal Matching Adds

The paired-interface method adds an engineering layer between conventional cycle schematics and complete-cycle thermodynamic comparison.

Paired-interface visibility. Conventional reservoir-only representations leave the thermodynamic-medium side of each heat-transfer interface implicit. Making both TM and ETS trajectories explicit reveals temperature ownership, local driving differences and process-wide trajectory behaviour.

Thermal Matching as an engineering degree of freedom. Thermal Matching improves local thermal alignment through deliberate changes in temperature level, trajectory shape, heat-duty distribution or combinations of these. Improvement attributable solely to increased heat-transfer conductance remains conductance enhancement. Non-isothermal interactions add trajectory-shape matching as a further design degree of freedom.

Finite-conductance and reversible limits. A better finite-conductance configuration need not have equal slopes, uniform local temperature differences or pointwise coincidence. The reversible local heat-transfer limit is the stronger condition in which the paired boundary-temperature trajectories coincide pointwise over the complete interaction.

Two-level assessment. Paired T-Q trajectories and local $\Delta T(Q)$ diagnose Thermal Matching or Thermal Mismatching at interface level. The thermodynamic work consequence is quantified at complete-cycle level by comparing the irreversible cycle with its corresponding reversible cycle; formal lost-work or excess-work-input quantification remains in the Formal Core.

This companion has demonstrated the method with heat pumps. The same paired-interface logic is applicable to heat engines, refrigeration cycles and other thermodynamic cycles with externally coupled heat-transfer processes. Broader application opportunities are addressed in the Impact, Applications & Research Opportunities Report; the formal corresponding-reversible-cycle construction and cycle-level lost/excess-work quantification are developed in the Formal Core.

Appendix A - Canonical Equations and Illustrative Data

Item	Fully isothermal	External-source glide	TM glide
High donor TM	60 °C constant	60 °C constant	60 °C constant
High receiver ETS	50 °C constant	50 °C constant	50 °C constant
High ΔT	10 K constant	10 K constant	10 K constant
Low donor ETS	10 °C constant	20 - 10q °C	10 °C constant
Low receiver TM	5 °C constant	5 °C constant	-5 + 10q °C
Low ΔT	5 K constant	15 - 10q K	15 - 10q K
Low dTETS/dq	0	-10 K	0
Low dTTM/dq	0	0	+10 K

A.1 Governing direct diagnostics

$$\Delta T_j(q) = T_{\text{donor},j}(q) - T_{\text{receiver},j}(q)$$

$$\text{reversible thermal interface} \Leftrightarrow \Delta T_j(q) = 0 \text{ for every } q$$

$$\text{linear profile slope: } \frac{dT}{dQ} = \frac{T_{\text{out}} - T_{\text{in}}}{Q_{\text{tot}}}$$

All temperatures used in thermodynamic ratios must be absolute temperatures in kelvin. Degrees Celsius are used only for the illustrative diagram labels because temperature differences in kelvin and degrees Celsius have equal numerical increments. No Carnot or Clausius ratio is evaluated from the Celsius values on this page.

NO UNIVERSAL SCALAR INDEX
The local gap and its heat-coordinate distribution are retained as problem-specific interface information. This report introduces no universal scalar Thermal Matching or thermodynamic-irreversibility index and does not assign a standalone lost-work magnitude to an individual finite-temperature-difference interface. The thermodynamic work consequence is treated at complete-cycle level through the corresponding reversible-cycle comparison in the Formal Core; local attribution requires an isolating comparison class.

Appendix B - Diagram Production Specification

B.1 Mandatory visual checks

Check	Acceptance requirement
Entity separation	TM and ETS are separate labelled objects at both interfaces.
One medium	The high and low TM stages are visibly connected by one cycle route.
Heat direction	QL: ETSL → TML; QH: TMH → ETSH.
Work direction	Win points into the cyclic system.
Temperature ownership	Every T label identifies the physical side to which it belongs.
Common coordinate	The two trajectories of one interface use the same Q or q axis.
Line semantics	External thermal system solid; thermodynamic medium dashed; isothermal horizontal; glide sloped.
Gap annotation	State whether ΔT is constant, varying or a minimum/maximum/terminal value.
Qualification	Caption distinguishes illustrative geometry from calculated or validated performance.
Accessibility	Meaning is available from labels and line styles, not colour alone.

B.2 Preferred figure caption pattern

CAPTION TEMPLATE
Figure X. [Cycle function and interaction structure]. [Donor and receiver trajectories]. [Direction and local temperature-difference interpretation]. [What is omitted]. [Whether values are illustrative, calculated or validated]. Source: [declared source].

B.3 File-production recommendation

Maintain a vector master for each diagram and export a high-resolution publication asset for Word. Store the numerical trajectory data and equation definition alongside the figure so that a static publication graphic remains reproducible even when it is not edited directly as Word drawing shapes.

Scientific Source Basis, Status and Rights Notice

Scientific source basis

Governing scientific source: Formal Core R185-L18, Operationalizing the Carnot and Clausius Reversible-Cycle Equalities for Process-Specific Thermodynamic Benchmarks. Governing terminology and positioning source: Scientific Positioning Standard v4.2/R181. Visual and programme-positioning reference: Impact, Applications & Research Opportunities Report v1.8. That report distinguishes established temperature-glide matching and T-Q profile analysis from the programme's proposed cycle-level integration. Contextual prior art includes Zühlsdorf et al. (2018), "Analysis of temperature glide matching of heat pumps with zeotropic working fluid mixtures for different temperature glides", Energy 153, 650-660, doi:10.1016/j.energy.2018.04.048. The present report therefore does not claim priority for temperature-profile matching itself. Its proposed contribution is a physics-explicit paired-interface visual and operational framing within the CarnotX programme, connected to the Formal Core's proposed complete operationalization of the Clausius reversible-cycle equality for constructing corresponding reversible cycles and quantifying benchmark-relative lost or excess work. The formal cycle construction and work-penalty calculation are not reproduced here. The three canonical model cases and their illustrative temperatures consolidate the author-directed working decisions developed in the present project thread.

Status

Item	Classification
A/A and B/B architecture codes	Proposed report-specific visual method; the preceding reservoir-only abstraction is conventional and is used as the comparison baseline.
Thermodynamic distinctions	Established science and Formal Core interpretation as identified in the text
Three numerical examples	Illustrative schematic cases; not equipment simulations
Thermal Matching integration	Proposed paired-interface cycle-level framing aligned with the Formal Core Thermal Matching definition. Temperature-glide/profile matching itself is established in specialist engineering literature and is not claimed as new here. The connection to corresponding reversible-cycle construction and benchmark-relative lost/excess-work quantification belongs to the broader Formal Core method.
Validation	Internal consistency and document QA only; no independent scientific or experimental validation claimed.

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