Scientific progress: extending the applicability of existing insights

From a Specific Formula to a General Method

The square–rectangle analogy for the proposed relationship between Carnot and Clausius

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Further information: *CarnotX Academy*

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Scientific progress: explaining more within one framework

A broader explanation can encompass existing knowledge and address new cases.

A broader scope

Scientific progress can occur when a framework explains more cases coherently. A more general framework can encompass the valid classical explanation as a specific case.

Connecting existing knowledge

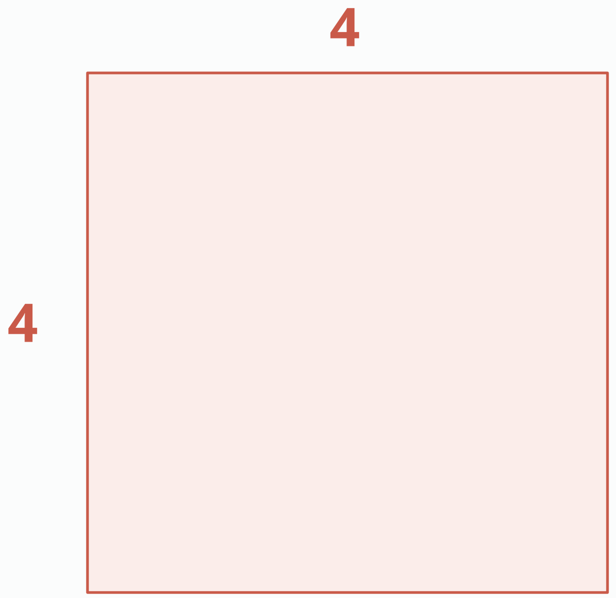
Individual special-case methods find a place within one general approach. This reveals their interconnections and provides a common basis for analysis and calculation.

The purpose of this presentation

To use the square–rectangle analogy to clarify the proposed relationship between Carnot and Clausius: what becomes more general, what remains valid and what requires assessment?

The square as the classical reference

The classical principle for the area of squares:
area = the side length squared



Four equal sides
Four right angles

a = side length
Area: $A = a^2 = 4^2 = 16$

For every square, exactly:
area = side length squared.

From square to rectangle: a broader formula

$A = a^2$ applies specifically to squares; a is the side length

Squares

Ratio 2 : 1

Ratio 3 : 1

a by a
$A = a^2$

$2a$ by a
$A \neq a^2$
Square formula not applicable

$3a$ by a
$A \neq a^2$
Square formula not applicable

Exact special-case methods for specific ratios

Each special-case method calculates the area exactly within its scope of applicability.

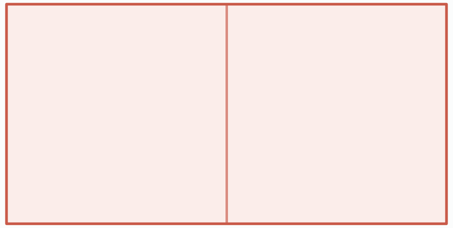
Squares

a by a

Area of square

$A = a^2$

Ratio 2 : 1



2a by a

Area = 2 × area of square

$A = 2 \times a^2$

Valid when length = 2 × width

Ratio 3 : 1

3a by a

Area = 3 × area of square

$A = 3 \times a^2$

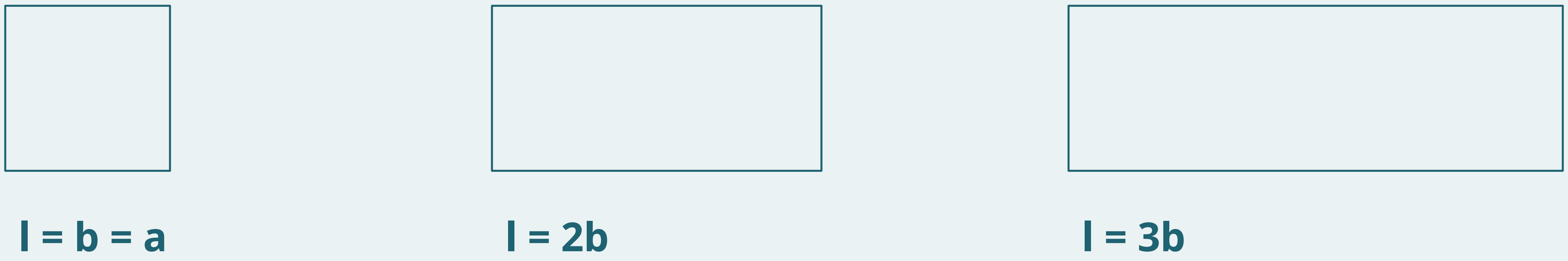
Valid when length = 3 × width

Exact for each ratio — no general formula for arbitrary length and width.

One general method for rectangles and squares

Area = length × width, for both rectangles and squares

Four right angles. Length l and width b.



Area A = l × b

An exact formula within a more general method

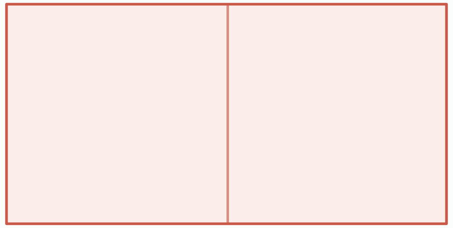
From a specific formula, through special cases, to one general method.

Specific formula

$A = a^2$

Exact for every square. The side length determines the area.

Exact special-case methods



$A = 2a^2$ or $A = 3a^2$

Dividing rectangles into equal squares yields exact results for specific ratios.

General method

$A = l \times b$

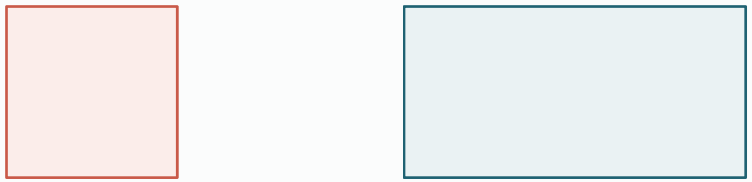
One method for every rectangle. The square is included: $l = b = a$.

The specific formula remains valid. The general method encompasses it and other ratios.

From area to efficiency

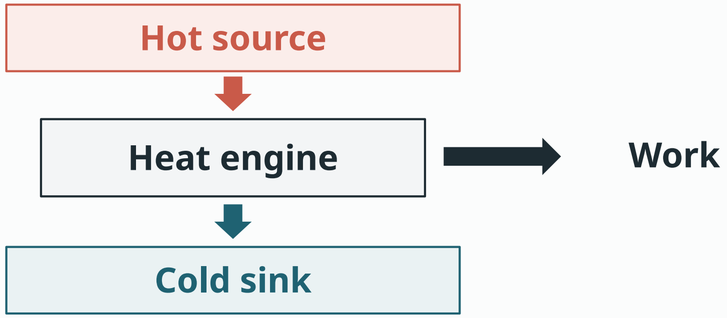
For squares and rectangles, we saw how a specific, exact formula fits within a more general method. With this in mind, we now examine thermodynamic cycles.

The geometric analogy



One formula for squares; a more general method for both rectangles and squares.

The thermodynamic question

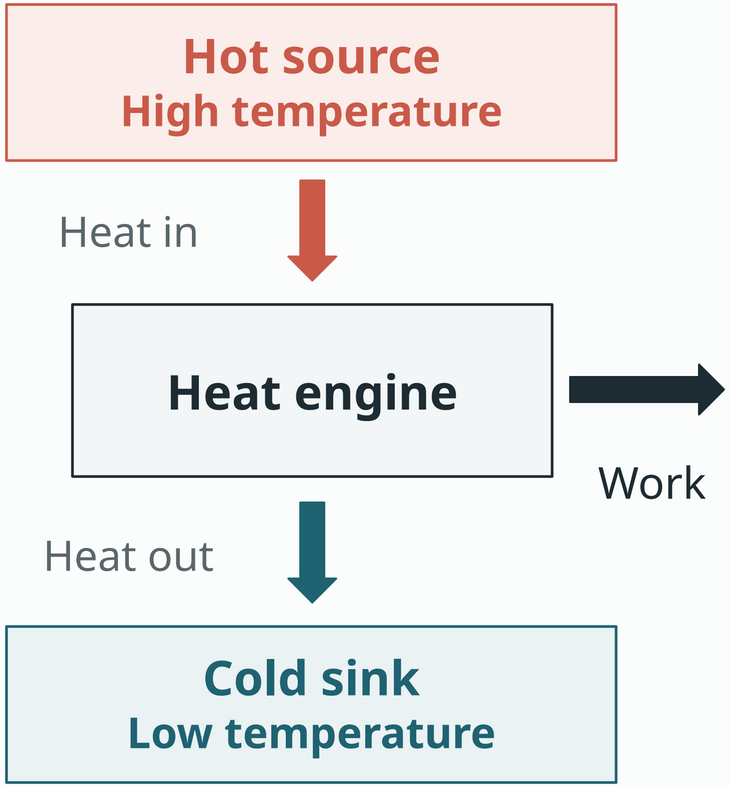


How does Carnot efficiency at two fixed temperatures relate to a general framework for reversible cycles, including those with varying temperatures?

First: how does a heat engine work, and what does efficiency mean?

How does a heat engine work?

Absorbing heat, delivering net work and rejecting heat



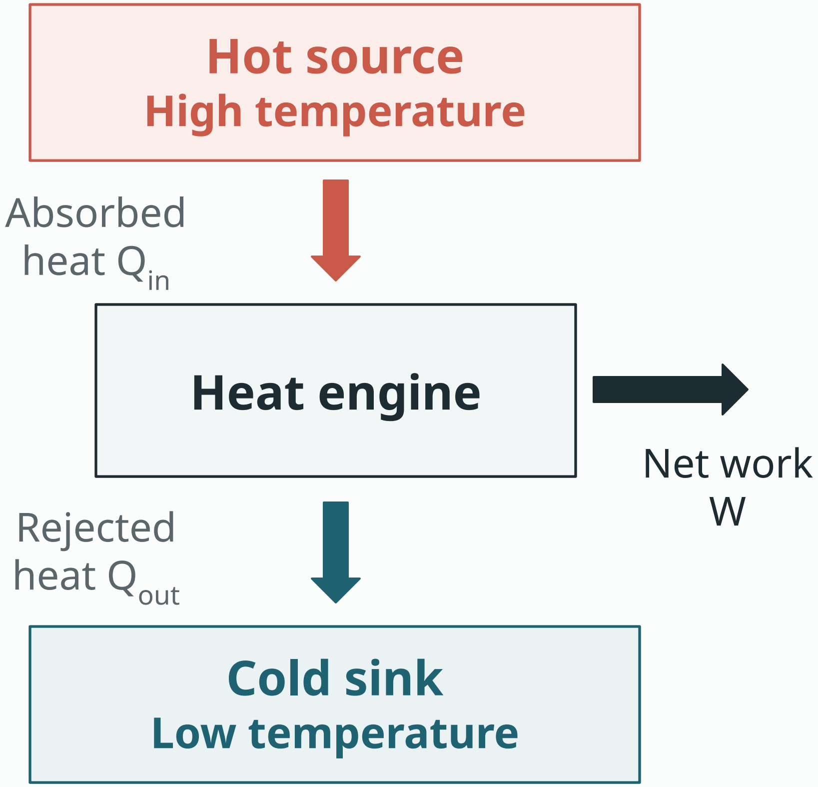
From heat to work

A heat engine absorbs heat, delivers net work and rejects the remaining heat.

During heat transfer from the **hot source** to the heat engine and from the heat engine to the **cold sink**, the temperatures may remain **constant** or **vary**.

What does heat-engine efficiency mean?

What fraction of the absorbed heat is converted into net work?



$$\eta = \frac{W}{Q_{in}}$$

Efficiency = net work delivered / heat absorbed.

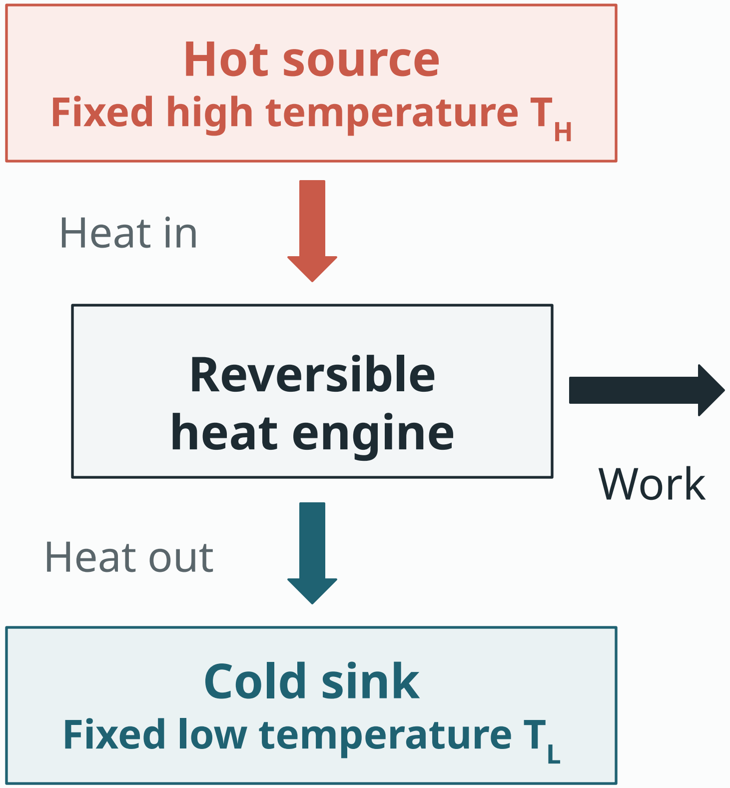
Efficiency indicates the fraction of the absorbed heat that becomes available as net work.

Example per cycle: 100 J heat in
40 J work + 60 J heat out
η = 40 / 100 = 40%

Valid for both constant and varying source and sink temperatures. The next slide addresses Carnot at two fixed temperatures.

Carnot: heat exchange at two fixed temperatures

Classification by external heat exchange: isothermal and non-isothermal cycles.



Carnot belongs to the class of cycles with isothermal heat exchange

Here, this means cycles with isothermal heat absorption and heat rejection: the temperature remains constant during each of these processes.

Heat absorption at T_H

The heat engine reversibly absorbs heat at the fixed high temperature.

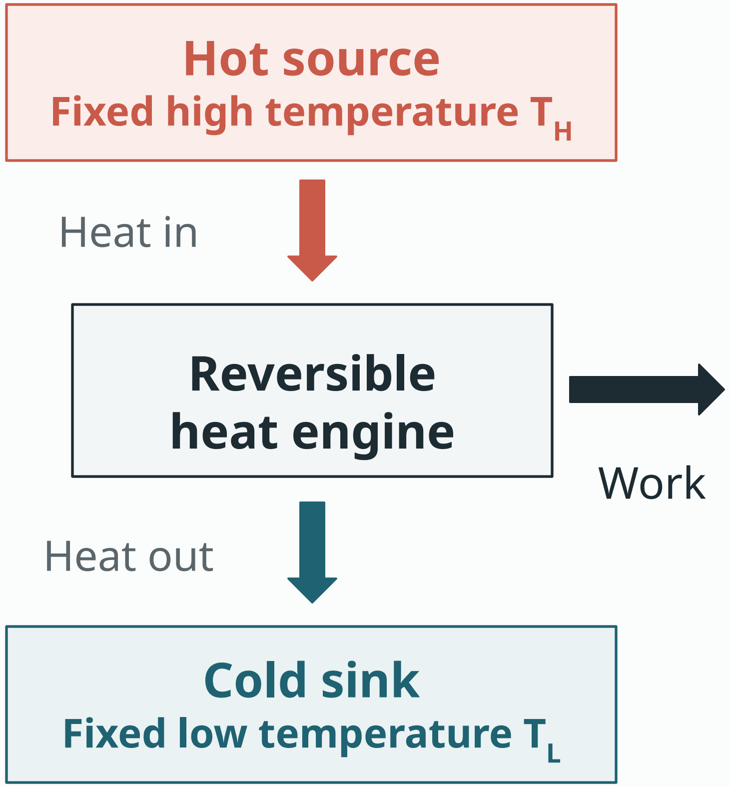
Heat rejection at T_L

The heat engine reversibly rejects heat at the fixed low temperature.

Between these two temperatures, the temperature changes during steps without heat transfer.

The classical Carnot efficiency

Attained exactly by a fully reversible heat engine operating between two fixed temperatures.



Temperatures in kelvin.

$$\eta_{\mathrm{Carnot}} = 1 - \frac{T_{\mathrm{L}}}{T_{\mathrm{H}}}$$

This efficiency is attained only by fully reversible heat engines with isothermal heat absorption and heat rejection at two fixed temperatures.

'Loss-free' means no irreversibilities in the cycle or its heat exchange. Over a complete reversible cycle, the Clausius integral equals zero. The loss of work potential calculated through exergy analysis is then also zero.

With irreversibilities, efficiency is lower. At the same hot-source and cold-sink temperatures, the formula remains the upper bound.

Cycle types: isothermal and non-isothermal heat exchange

Classification by the cycle’s external heat absorption and heat rejection.

Isothermal external heat exchange

Carnot
Stirling*
Ericsson*

Non-isothermal external heat exchange

Brayton · Otto
Diesel · Atkinson
Kalina · Rankine

Each cycle has **its own formulas** for efficiency, valid under the **chosen assumptions**.

* Ideal Stirling and Ericsson cycles with perfect regeneration.
The shapes indicate categories; they are not thermodynamic diagrams.

A specific formula is not a general calculation method

Square and rectangle

$$A = a^2$$

Specific to squares

Exact for a square with side length a.

When length and width differ,
a^2 does not determine the area.

Carnot and non-isothermal cycles

$$\eta_{\mathrm{Carnot}} = 1 - \frac{T_{\mathrm{L}}}{T_{\mathrm{H}}}$$

Exact within this scope

Exact for reversible heat transfer at two fixed temperatures.

The highest and lowest temperatures alone do not determine a reversible reference for the specific non-isothermal cycle.

The formula remains exact within its scope of applicability.

How can we also describe the other configurations exactly?

The proposed relationship between Carnot and Clausius

From a specific, exact formula to an overarching framework for comparison

Clausius: general framework for comparison

Carnot
Stirling*
Ericsson*

Isothermal cycles: two fixed temperatures

Encompasses isothermal and non-isothermal cycles.

Also at multiple or varying temperatures.

* Ideal Stirling and Ericsson cycles with perfect regeneration.

Established foundation

For a reversible cycle:

$$\oint \frac{\delta Q_{\mathrm{rev}}}{T} = 0$$

Proposed operationalization

Develop this equality, together with the energy balance and the specified heat-exchange trajectory, into a generally applicable benchmark method.

No new law of nature

From reversible cycle to efficiency

Carnot: two fixed temperatures

Established derivation

For reversible heat exchange at two fixed temperatures.

$$\eta_{\mathrm{rev}} = f\left(\frac{Q_{\mathrm{in}}}{T_{\mathrm{H}}} - \frac{Q_{\mathrm{out}}}{T_{\mathrm{L}}} = 0\right)$$

This gives:

$$\eta_{\mathrm{rev}} = 1 - \frac{T_{\mathrm{L}}}{T_{\mathrm{H}}}$$

From equality to efficiency

The step via f

The equality describes reversible heat exchange.

The energy balance links heat to work.

The full mathematical derivation yields the efficiency.

f denotes the full mathematical operationalization of the equality, including the energy balance, specified heat exchange and boundary conditions.

Q_{in} and Q_{out} are positive quantities of heat. T_b is the local absolute temperature where heat crosses the cycle boundary.

Efficiency is one application within the proposed general framework for reversible cycles.

From reversible cycle to efficiency

Carnot: two fixed temperatures

Established derivation

For reversible heat exchange at two fixed temperatures.

$$\eta_{\mathrm{rev}} = f\left(\frac{Q_{\mathrm{in}}}{T_{\mathrm{H}}} - \frac{Q_{\mathrm{out}}}{T_{\mathrm{L}}} = 0\right)$$

This gives:

$$\eta_{\mathrm{rev}} = 1 - \frac{T_{\mathrm{L}}}{T_{\mathrm{H}}}$$

Clausius: also at varying temperatures

Proposed operationalization

Also for reversible heat exchange at varying temperatures.

$$\eta_{\mathrm{rev}} = f\left(\oint \frac{\delta Q_{\mathrm{rev}}}{T_{\mathrm{b}}} = 0\right)$$

f denotes the full mathematical operationalization of the equality, including the energy balance, specified heat exchange and boundary conditions.

Q_{in} and Q_{out} are positive quantities of heat. T_b is the local absolute temperature where heat crosses the cycle boundary.

Efficiency is one application within the proposed general framework for reversible cycles.

From thermodynamic foundations to the proposed method

Carnot:
specific reference

Carnot efficiency is attained exactly with fully reversible heat exchange at two fixed temperatures.

Clausius:
general foundation

The Clausius equality for reversible cycles also applies to non-isothermal heat exchange.

Proposed
operationalization

Develop this equality, together with the energy balance, heat exchange and boundary conditions, into a generally applicable benchmark method.

Carnot is retained as a specific case within the proposed general framework for reversible cycles.

What implications could this have for education and engineering if the method is independently confirmed and offers added value?

Potential impact on education and engineering

If the proposed method is confirmed and offers demonstrable added value,
its implications could be wide-ranging.

Education and technical literature

The proposed modernization of thermodynamic cycle theory could affect the content of more than 300 unique book titles.

Preliminary estimate within this project; impact depends on validation.

Analysis and design

Could help engineers target energy-efficiency optimization more effectively: combustion engines, power plants, nuclear power plants, heat pumps and refrigeration systems.

Added value to be assessed for each application.

The Lorentz workshop: independent assessment

Three assessment questions: independent reconstruction, comparison and delimitation of scope.

1 Is the derivation correct?

Independently reconstruct and verify.

Make assumptions and the limits of the method’s validity explicit; search for counterexamples.

2 What do the methods add?

Compare with existing methods.

Examine numerical examples for isothermal and non-isothermal cycles. Identify additional insights.

3 Which implications can be demonstrated?

Substantiate what the results mean for education and technical literature.

Assess the implications for analysis and design and define the remaining open questions.